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Dedicated to the memory of
Professor Cristofor I. Simionescu (1920–2007)

COHERENCE RESONANCE IN EXCITABLE SYSTEMS: ACTIVATOR-INHIBITOR DYNAMICS

Cristina STAN,[a*] Constantin P. CRISTESCU[a] and Dumitru ALEXANDROAEI[b]

[a]"Politehnica" University of Bucharest, Faculty of Applied Science, Department of Physics I, 313 Spl. Independenței, RO-060042, Bucharest, Roumania
[b]"Al.I.Cuza" University of Iaşi, Faculty of Physics, 11 Carol Blv., Iaşi, RO-700506, Roumania

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In this paper we report on a special type of stochastically induced dynamics in an exitable system consisting of an electrical charge double layer configuration, as usually found in chemical systems, biological cell membranes, electrical plasmas and nanostructures. A Gaussian noise can induce almost regular dynamics without any periodic signal injected into the system. The inter-spike statistics presents a clear maximum as function of the noise amplitude, behavior known as coherence resonance. In order to theoretically describe this behavior, we derived an excitable system by modifying a biased van der Pol oscillator, known to suitably model a double layer structure in oscillatory regime. The computational study considers the behavior of this system under the influence of externally injected Gaussian noise. The model is found to well reproduce the experimentally observed dynamics for a charge double layer generated in the inter-anode space of a twin electrical discharge subject to noise perturbation.

INTRODUCTION

Recently, there is a growing interest in the field of excitable systems driven by noise. A generic excitable system has specific temporal behaviour expressed in terms of excitability, the existence of a threshold and the development of oscillations. Subthreshold perturbations from the steady state return the dynamics of the system to the steady state, while superthreshold perturbations have a large transient excursion before returning the system to the steady state. A particularly well-studied example of excitable system is described by two variables whose dynamics is an activator-inhibitor type. The activator variable catalyzes its own production (autocatalysis) and also activates the formation of the inhibitor while this variable inhibits the formation of the activator. Details on the excitable systems dynamics can be found in previous works.[1,2] The simplest system showing activator-inhibitor dynamics is a charge double layer configuration.

It is well established that noise can induce quasiregular dynamics in nonlinear systems without the necessity of injecting a periodical signal from outside, behaviour known as autonomous stochastic resonance (ASR).[3,4] A phenomenon of this kind, induced by the system's own chaotic dynamics that plays the same role as noise in conventional stochastic resonance was recently reported.[5,6] A somewhat similar behavior is shown by excitable systems. The transition from steady state to nearly periodical dynamics in an excitable system under the influence of noise and in the absence of any external periodical perturbation is known as coherence resonance (CR).[7-9]

In this paper, we report on the experimental investigations of the stochastically induced dynamics in a double layer (DL) electrical charge configuration as paradigm of activator-inhibitor dynamics. The DL is generated in the inter-anode space of a twin electrical discharge plasma. This discharge arrangement is suitable for the study of the DL dynamics due to its capacity of controllably and reproducibly generating DLs in selected

* Corresponding author: cstan@physics.pub.ro

dynamical regimes by changing the inter-anode biasing. The theoretical interpretation is based on an excitable system obtained by modifying a biased van der Pol oscillator. The results of the computational study on the behavior of this system under the influence of externally added Gaussian noise is compared with the experimental results.

The relevance of this study consists in the fact that the DL electrical charge structure is a fundamental configuration found in many chemical systems,[10,11] biological cell membranes[12,13] and nanostructures.[14] This work is also relevant for plasma supported chemical reaction. As an example, we mention chemical synthesis in different types of gaseous mixtures using cold plasma generated by electric discharges as an energy source. However, the spectrum of interests is very large, ranging from the generating of small organic compounds existing in the living world starting from a few small inorganic compounds[15] to plasma assisted catalysis with applications from industry to the control of polutants in gaseous phase wastes.[16]

EXPERIMENTAL

A sketch of the experimental device is shown in Fig. 1. Details on the system are presented elsewhere.[17]

Two independent electric discharges at low pressure (80 mTorr) in flowing Argon are running between the electrodes K1-A1 and K2-A2 respectively, placed in the same glass tube. A dc voltage source maintains a constant biasing U of one anode against the other. A perturbed regime can be generated if a small variable voltage is connected in series with the dc biasing. In the present study, the perturbation is generated by a Gaussian noise supply (denoted U_n on Fig.1), with an equivalent standard deviation U_s=10Vrms, coupled to the discharge through an attenuation network. The dc biasing U generates the DL space charge structure while U_n perturbs it and controls its dynamics. The DL is the source of oscillations in the inter-anode plasma and its behavior can be efficiently controlled by the characteristics of the biasing.[18,19]

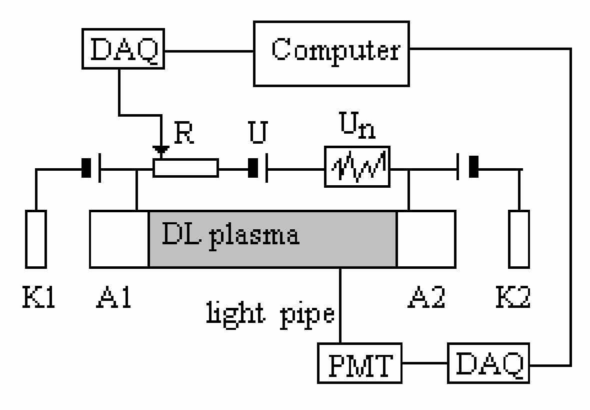


Fig. 1 – Sketch of the experimental set-up.

In this study, we are interested in a small range of inter-anode biasing in the neighborhood of the threshold where a transition between steady state and periodical dynamics is taking place. From the dynamical point of view, a limit cycle can disappear if the control parameter (related to the biasing voltage) is adjusted below a critical value where the DL exists in a steady state (supercritical Hopf bifurcation). Below threshold, the intrinsic oscillation is only present as a transient behavior corresponding to the evolution of the system towards its steady state after some perturbation. The addition of Gaussian noise to the system in the steady state can induce the transition towards an oscillatory dynamics. For a particular noise level, the induced dynamics shows an optimum degree of coherence.

We study the dynamics of the DL as reflected in the temporal behavior of the current flowing through the inter-anode space. The local light intensity recorded by a photomultiplier (PMT) placed in the neighborhood of the DL region is also recorded. This arrangement gives us the possibility to investigate correlations between the temporal variations of the DL parameters and the temporal variations of plasma global ones (the inter-anode current).

Fig. 2 shows the simultaneous acquisition of the light intensity in the DL region (3), the inter-anode current (2), and the temporal variations of the applied noise voltage U_n (1) for an optimum amplitude added noise. The correlation between the light and the current oscillations demonstrates that the DL dynamics in the oscillatory regime induced by the noise generates the same kind of behavior in the whole inter-anode plasma. We consider this correlation as good justification for the further use of the current in the present investigation. It should be emphasized that under the same circumstances, but in the absence of the noise, the traces (3) and (2) are almost flat, showing only low amplitude system fluctuations.

We observe a clear dependence of the regularity in the current fluctuations on the noise level. Maximal regularity is observed for particular noise amplitude demonstrating coherence resonance. This effect can be characterized by various measures,[1,20] one of the most popular, the one that we use in the present work, being the inter-spike statistics $\langle T \rangle / \sigma$. Here, $\langle T \rangle$ is the average period calculated by counting the number of maxima in a long time interval and dividing the length of the interval to the corresponding number of maxima and σ is the standard deviation on this period.

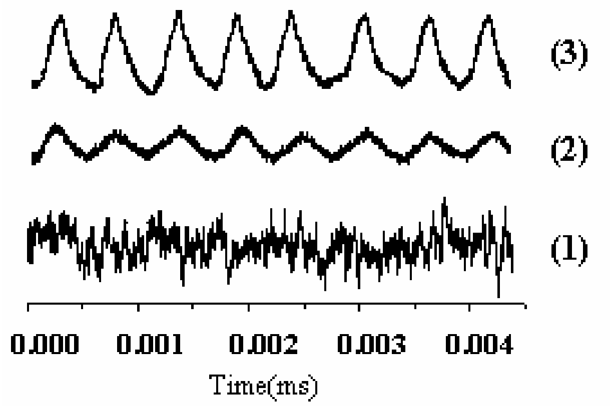


Fig. 2 – Correlation between the local light signal (3) and the inter-anode current (2) in the presence of the applied noise (1).

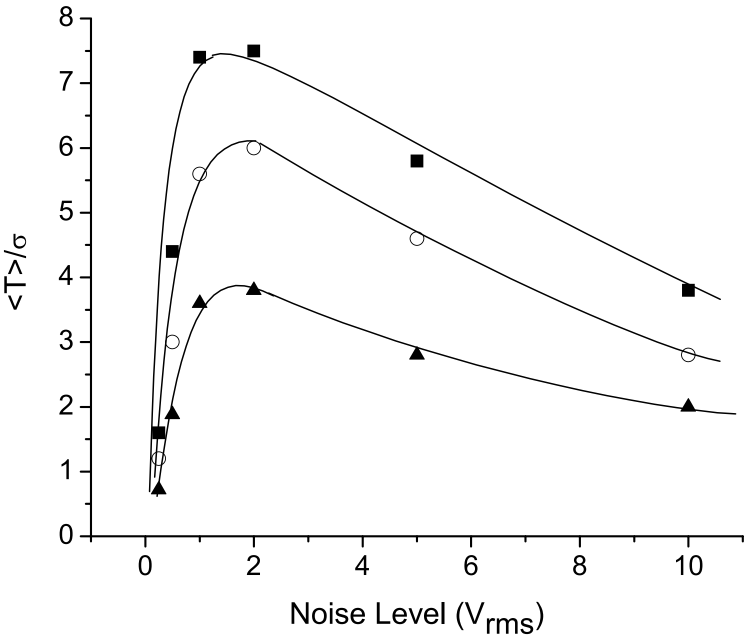


Fig. 3 – Curves of the experimental inter-spike statistics $\langle T\rangle / \sigma$ versus the noise level for the values of the dc biasing: 17V (triangles); 17.5V (open circles) and 18V (squares).

Fig. 3 presents the inter-spike statistics for three values of the dc inter-anode biasing in the range between 17-18V. The continuous lines are drawn for eye guiding only. The curves present a clear maximum, characteristic of CR.

Stochastic resonance-like behavior in electrical discharges was previously reported in a weakly ionized radio frequency magnetoplasma[21] as spontaneously generated nonlinear ionization drift waves and in a simple configuration electrical discharge in neon[22] as waves of stratification generated by a convective instability of the positive column.

EXCITABLE SYSTEM MODEL AND COMPUTATIONAL RESULTS

The generation of a charge double layer, irrespective of the physical system, involves electrical potential biasing. As is well known, the oscillatory dynamics of a DL is well described by the van der Pol equation.[23] Accordingly, we begin our analysis with the biased van der Pol equation:

$$\ddot{y} - b\dot{y}(1-y^2) + cy = n\,. \tag{1}$$

By introducing a Liénard variable[24] this can be cast into a system of two first order equations:

$$\begin{aligned} \dot{x} &= ay + m \\ \dot{y} &= b\left(y - \frac{y^3}{3}\right) - dx + k \end{aligned}\,. \tag{2}$$

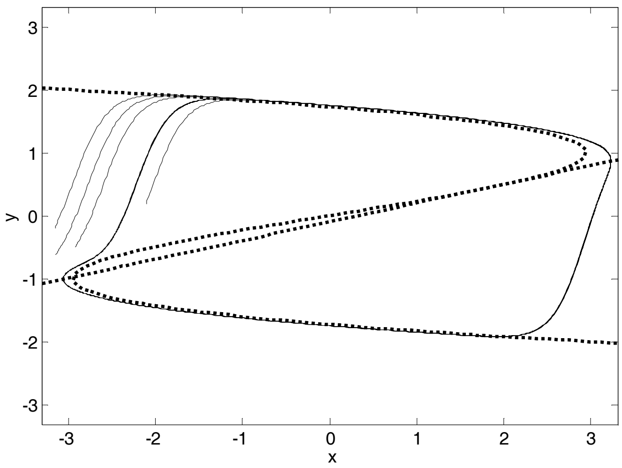
As observed from experiment, without the biasing, the DL system does not exist. For this reason, we have to phenomenologically introduce in the first equation a damping term ($-gx$). In the same equation, we also introduce a time depending perturbing term ($f(t)$) that corresponds to the additional biasing. We end up with the system:

$$\dot{x} = ay - gx + m + f(t)$$
$$\dot{y} = b\left(y - \frac{y^3}{3}\right) - dx + k \, . \qquad (3)$$

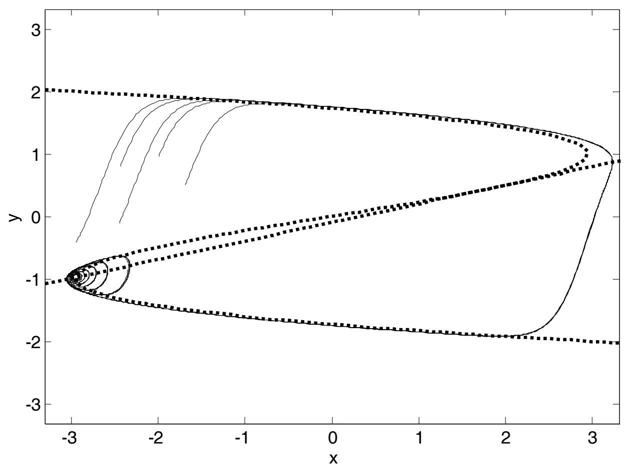
For the sake of simplicity, in the computation we take k=0. As expected, the system is similar to the well known FitzHugh-Nagumo (FH-N) model or the Bonhoeffer van der Pol (BvP) oscillator. Consequently, we expect the system (3) to properly describe the excitable regime of the DL configuration. We investigate the dynamics induced by an externally applied Gaussian noise $f(t) = D\xi(t)$, where $\xi(t)$ is the Gaussian noise function characterized by zero mean and delta correlation:

$$< \xi(t)\xi(t') >= \delta(t - t') \, . \qquad (4)$$

For a wide range of the parameters and in the absence of noise, the system (3) presents two distinctive dynamics depending on the values of the control parameter m. Below some threshold value, the system presents periodic oscillations while above that value, subsequent to some perturbation, it evolves by damped oscillation towards a stationary state after a large excursion (excitable behavior). Fig. 4 shows phase portraits of the system (3) without noise for the values of the parameters indicated in the caption of the figure: limit cycle oscillation (a) and stationary state (spiral sink) (b). The broken lines represent the nullclines of the system.



a)



b)

Fig. 4 – Phase portraits of the system (3) without noise (D=0) for the following values of the parameters: a=1; b=4.9; d=1.1; g=0.3; a) limit cycle oscillation for m=0.098; b) stationary state (spiral sink) for m=0.1.

There are two main differences between the system (3) and the FH-N or BvP systems. The first one is a consequence of the dissimilarity of the physical structures they model and is reflected in the rates of change of the two variables. Usually, the FH-N and BvP equations are used to describe the dynamics of neuronal response to a stimulus. If a short perturbation removes the system from its stationary state, then, a fast variable (in this case, the membrane voltage) keeps growing until it reaches a certain excited state called "firing state". Another variable, the recovery variable acting on a slower time scale destabilizes the excited state of the voltage variable bringing it back to the rest state. The representative point travels through a large cycle in phase space.

For a different value of some control parameter, the dynamics of the system consists in periodic spiking. This is a strongly nonlinear behaviour, consequence of the disparity in the rates of change of the two variables.

Unlike this, in the oscillatory regime, the DL structure generates a slightly nonlinear periodic oscillation that can be accounted for only if the rates of change of the two variables are sensibly equal.

The second difference can be observed from the diagrams in Fig. 4. While the order three nullclines of the FH-N and BvP systems are of inverted N-shape or N-shape, in our system, the shape of the third order nullcline is an inverted S. This entails a richer spectrum of dynamics.

Fig. 5 shows the behavior of the model with respect to the level of the injected noise, corresponding to the experimental situations shown in Fig. 3. A clear agreement with the experimental curves obtained for values of *U* below the oscillation threshold is observed.

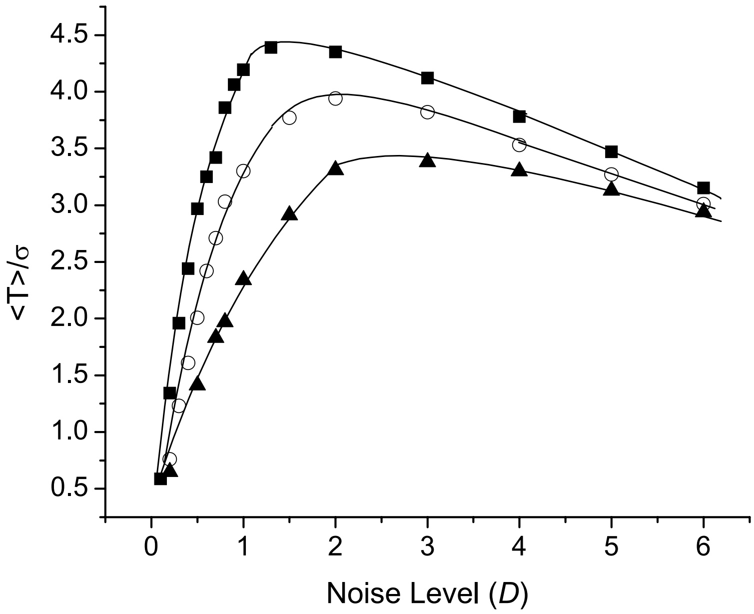


Fig. 5 – Results of the computational model corresponding to the experimental curves in Fig. 3; Curves of the inter-spike statistics $\langle T \rangle / \sigma$ versus the noise level computed for a=1; b=4.9; d=1.1; g=0.3 and for the values of the control parameter m: 0.17 (triangles); 0.14 (open circles) and 0.11 (squares).

CONCLUSIONS

The activator-inhibitor dynamics of an excitable system represented by a DL structure has been studied both experimentally and theoretically. The behavior of this charge configuration is changing as the inter-anode biasing transcends a certain threshold value, going through a transition from stationary state to oscillatory dynamics (supercritical Hopf bifurcation). We demonstrate that such a system in the stationary state can be forced into quasiregular dynamics by injecting Gaussian noise. The curves of the inter-spike statistics versus the noise level show a well defined maximum, characteristic of coherence resonance. The theoretical treatment is based on a system of equations fit to model an excitable system, derived by modifying the biased van der Pol oscillator. The good agreement between the experimental data and the computational treatment demonstrates that this system represents a suitable model for the activator-inhibitor behaviour. The study of the dynamics of this simple excitable system can have relevance to many fields of science and technology, from chemistry and cell biology to nanostructures since charge double layers characterize many complex microscopic configurations.

Chemical reactions showing excitability-like behaviour are reasonably common. One of the best known is the nonlinear chemical oscillator known as the Belousov-Zhabotinsky (BZ) reaction. There are many BZ mixtures reported in the chemical literature, the common element in these oscillating systems being their property of taking place far from thermodynamic equilibrium.[24] The chain of individual chemical reactions is very long. It can be written in a system of about a dozen equations and after serious simplification it can be reduced to a system of two equations similar to our model (2).

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