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ANALYSIS ON THE THERMAL BEHAVIOUR OF THE THERMAL STORAGE UNITS FOR PHASE CHANGE MATERIALS

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The promotion of the unconventional heating/cooling systems of dwelling/occupied spaces requires, in most cases, their equipping with heat storage units either in a sensitive environment or in phase change substances. The sizing of the systems components as well as the assessment of the energy efficiency in representative climatic conditions is an activity requiring knowledge of the Thermal Storage Units Thermal Response (TSU-TR) at the loads represented both by the climatic parameters variation and by the dwelling exigencies in occupied spaces. The basic problem in sizing the PCSU (Phase Change Storage Units) is the accumulation of the maximum heat/cold quantity in a time-lag associated to the capacity of the source to supply heat/cold. The storage units fall into three types of geometry, namely: plane; cylindrical; spherical. The fluid carrying the heat/cold (heat carrier) has a very low thermal capacity (the mass is limited by avoiding the sensitive thermal processes (detrimental in terms of energy as well as in terms of sizing). Two types of processes are considered, as follows: with temperature of the heat carrier variation in time and space and isothermal at value t_F.

INTRODUCTION

One of the fundamental provisions of European Directive 31/2010/UE of 19 May 2010 concerning the buildings energy performance, which is the object of Art. 9, is represented by the performance of buildings with an energy consumption close to zero. This very severe target imposed on buildings can only be reached by efficiently using the energy alternative sources. In Roumania, solar energy may be used in thermal form in applications related to spaces conditioning.[1] The intermittent nature of the solar generated energy, on one hand, and the temperature level specific to conditioning thermal processes, on the other, require the performance of passive and/or active solar radiation collecting systems and of systems of conversion into heat which include Thermal Storage Units (TSU). One of the difficult issues in dimensioning such systems is to reduce the volume of such equipments. One of the solutions is the use of phase change materials as heat storage environment, in the form of Phase Change Thermal Storage Units (PCTSU). The PCTSU dimensioning requires knowledge of the phase change front shift, both in the thermal loading process and in the thermal unloading process, according to the thermal load profile associated to the conditioning processes in occupied spaces. The presence of the mobile boundary between the solid phase and the liquid phase associated to the phase change temperature constant value is specific to the phase change processes analysis. At this level, the boundary conditions specific to each of the two previously mentioned phases are formulated; in association with the heat transfer equations specific to the environments in contact, they generate the Stefan problem. The accuracy and the applicability of the solutions are verified in this report by the experimental validation procedure. The known

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solutions of the Stefan problem may fall into a few classes, most of them belonging to the numerical solutions class. The von Neumann[2] classical solution is representative for the recurrent analytical solutions class which starts from a function of the variation in time of the phase change front and is particularized by the identification of a constant specific to each problem. The enthalpy method[3] is based on defining the dimensionless enthalpy function as a replacement of temperature in the parabolic equation of conduction. A class of solutions with applicability in the range of the reduced values β (or Ste) is that obtained by Pedroso and Domoto using the perturbed parameter method.[4,5] An exhaustive analysis of the known numerical solutions is presented by Caldwell and Kwan.[6] All the previously mentioned solutions are numerical, not experimentally validated, which represents a barrier in working out a new and unitary method of dimensioning the PCTSU. The present report uses the principles of the perturbed parameter, support for one modified perturbed parameter method the object of which is the integration of the Stefan equation. This report presents the experimental validation of the calculation models based on experiments of in vitro type specific to various systems and various geometries of PCTSU. The control parameters are the moment when the phase change process is over and the heat quantity accumulated in the heat storage unit. The thermal response of the heat storage units with elementary geometries – plane, cylindrical and spherical – is presented as well as that of the heat storage units formed of spherical components crossed by a fluid with either a random variable inlet temperature (the case of storage units included in solar systems) or constant. The solutions found allow the dimensioning of the systems including low temperature heat storage units associated to the processes specific to the energy efficient buildings.[7] This report presents the results, in a succession of complex mathematical models, simplified mathematical models, experimental validation, with reference to the active systems of using energy alternative sources. The analytical calculation methods may be similarly applied to passive systems, where the isothermal fluid is replaced by the equivalent temperature of the solar radiation collecting unit.[7]

MATHEMATICAL MODEL OF THE PHASE CHANGE PROCESS

1. Mathematical model of the phase change process specific to the plane geometry for a heat carrier with a variable temperature

The fluid operating as a heat carrier is flown at a constant flow-rate among the plane units made of 2Δ thick phase change material. The phase change material is initially solid and its temperature is equal to the phase change temperature, t_F. As a consequence of the heat flow rate yielded by the heat carrier, the solid phase turns into liquid and the liquid phase temperature increases over value t_F. The modification of the liquid phase temperature occurs following the variation of the liquid phase internal energy and is expressed as function $\vartheta(y,\tau)$ for each value of coordinate $x > 0$. A simplifying hypothesis, easily used in practice by heat storage units dimensioning is to consider heat transfer in the liquid phase structure as by conduction. The necessary and sufficient condition is $Bi_F < 1$. We note that the model can also be applied if $Bi_F > 1$ for materials in the case of which the liquid phase solidification is associated to the appearance of a reticular network which annuls the manifestation of the natural convention currents in liquid phase (*e.g.* maleic anhydride – $C_4H_2O_3$). The mathematical model of the heat transfer between the fluid and the phase change material is formed of the heat carrier energy balance equation associated to the equation of the heat transfer by conduction through the liquid phase of the phase change material. The heat transfer specific to the phase change substance is defined by the Stefan problem where the position of the phase change front $Z(\tau)$ varies according to the time, for each x coordinate. The solution of the problem is to determine the phase change front position variation speed in order to estimate the moment when either the thermal loading or the thermal unloading of the PCTSU is completed. According to the source energy availability and of the user's energy-related profile, PCTSU and the system components are dimensioned so as to meet the technical or/and economic criteria justifying its actual performance. The report presents the mathematical model of the modified perturbed parameter method. The modification is generated by the association of Stefan equation to the thermal balance equation of the heat carrier and by the limiting of the solution defining series (first order) of number *Ste*. This

essential simplification is sustained by the low values of *Ste* dimensionless number in the case of low temperature applications specific to buildings. A second simplification specific to the calculation model presented is to overlook the variation of the phase change materials internal energy as this is in general under 10% of the total energy stored/yielded in the phase change processes. The solutions further presented are sustained by the experimental validations on full-scale models the results of which are presented in this report.

The process scheme is presented in Fig. 1.

The heat carrier thermal balance equation is:

$$\frac{\mathrm{d}t(x,\tau)}{\mathrm{d}x}+\frac{2\alpha_F}{G\cdot c}\cdot[t(x,\tau)-\vartheta(y,\tau)\big|_{y=\Delta}]=0 \qquad (1)$$

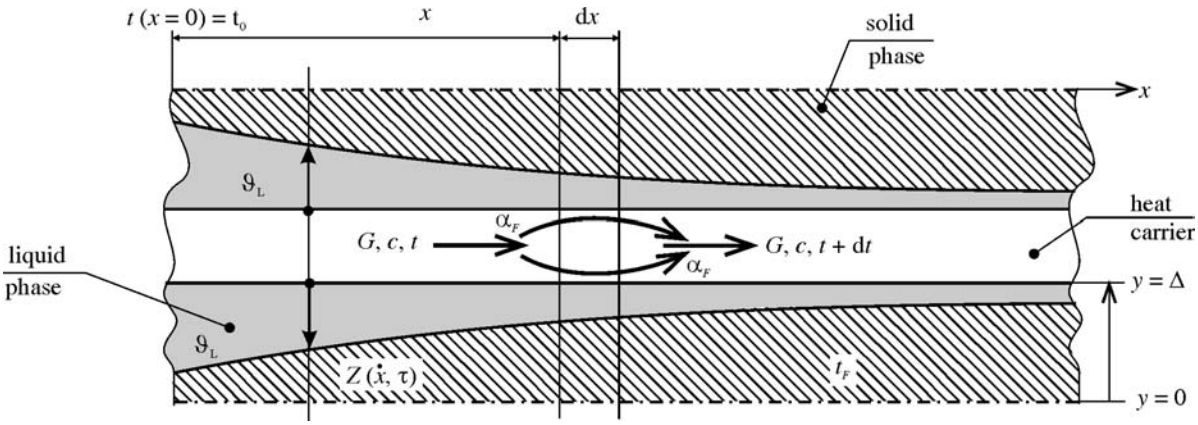

Fig. 1 – PCSU calculation scheme – plane geometry .

Modeling hypotheses:

1. At moment $\tau = 0$, the temperature of the phase change has a constant value, $\vartheta(y, \tau = 0) = t_F$.

2. The liquid state does not allow convection currents in the phase change substance (mainly because of the reduced sizes of the PCSU) – the heat transfer in the liquid mass is generated by conduction:

$$\frac{\partial\vartheta(y,\tau)}{\partial\tau}=a_L\cdot\frac{\partial^2\vartheta(y,\tau)}{\partial y^2} \qquad (2)$$

The boundary condition at the $y = Z(\tau)$ boundary between the substance in liquid state and that in solid state is provided by the Stefan equation:

$$\frac{\mathrm{d}Z(\tau)}{\mathrm{d}\tau}=-\frac{\lambda_L}{\rho_L\cdot L}\cdot\frac{\partial\vartheta(\tau)}{\partial y}\bigg|_{y=Z(\tau)} \qquad (3)$$

The Newton type boundary condition at the boundary between the heat carrier and the phase change material (PCM):

$$\alpha_F\cdot[t-\vartheta(y=\Delta,\tau)]+\lambda_L\cdot\frac{\partial\vartheta(y,\tau)}{\partial y}\bigg|_{y=\Delta}=0 \qquad (4)$$

The IVth type boundary condition at the phase change boundary:

$$\vartheta(y=Z,\tau)=t_F \qquad (5)$$

With dimensionless temperature $\Theta(y,\tau)=\dfrac{t_0-\vartheta(y,\tau)}{t_0-t_F}$ the problem is re-expressed as follows in terms of the new variable $\Theta(y, \tau)$:

$$\begin{cases}\beta\cdot\dfrac{\partial\Theta}{\partial Z}\cdot\dfrac{\partial\Theta}{\partial y}\bigg|_{y=Z}=\dfrac{\partial^2\Theta}{\partial y^2}\\ \alpha_F\cdot\left[\Theta(y=\Delta)-\dfrac{t_0-t}{t_0-t_F}\right]+\lambda_L\cdot\dfrac{\partial\Theta}{\partial y}\bigg|_{y=\Delta}=0\\ \Theta(Z,Z)=1\end{cases} \qquad (6)$$

where $\beta = \frac{c_L}{L} \cdot (t_0 - t_F) = \mathrm{Ste}$. The solution in the following form is adopted:

$$\Theta(Z, y) = \sum_{j=0}^{n} \beta^j \cdot \Theta_j(Z, y) \tag{7}$$

With the new dimensionless variables:

$$\dot{y} = \frac{y}{\Delta}; \quad \dot{Z} = \frac{Z}{\Delta}; \quad \mathrm{Bi}_F = \frac{\alpha_F \cdot \Delta}{\lambda_L} \tag{8}$$

The linear variation $\Theta = \Theta(\beta)$ is preserved as a representative solution:

$$\Theta(\dot{Z}, \dot{y}) = \Psi_1(\mathrm{Bi}_i, \dot{y}, \dot{Z}) + \Psi_2(\mathrm{Bi}_i, \dot{y}, \dot{Z}) \cdot \frac{t - t_0}{t_0 - t_F} + \beta \Psi_3(\mathrm{Bi}_F, \dot{y}, \dot{Z}) \cdot \left(1 - \frac{t_0 - t}{t_0 - t_F}\right)^2 \tag{9}$$

The heat carrier thermal balance equation is expressed as follows:

$$R = R_1 \cdot R_2 \cdot \frac{1 - \exp\left[-\varphi_2(\mathrm{Bi}_F, \dot{Z}, \beta) \cdot (R_1 - R_2) \cdot \dot{x}\right]}{R_2 - R_1 \cdot \exp\left[\varphi_2(\mathrm{Bi}_i, \dot{Z}, \beta) \cdot (R_1 - R_2) \cdot \dot{x}\right]} \tag{10}$$

with the following notations:

$$R = \frac{t - t_0}{t_0 - t_F}; \quad \dot{x} = \frac{x}{\ell} \tag{11}$$

where R_1 and R_2 are the solutions of the algebraic equation:

$$\beta \Psi_3(\mathrm{Bi}_i, \dot{Z}, 1) \cdot R^2 + [1 + \Psi_2(\mathrm{Bi}_F, \dot{Z}, 1) + 2\beta \Psi_3(\mathrm{Bi}_F, \dot{Z}, 1)] \cdot R + \Psi_1(\mathrm{Bi}_i, \dot{Z}, 1) + \beta \Psi_3(\mathrm{Bi}_F, \dot{Z}, 1) = 0 \tag{12}$$

We define: $\Psi_3(\mathrm{Bi}_i, \dot{Z}, 1) = \beta^{-1} \varphi_2(\mathrm{Bi}_F, \dot{Z}, \beta)$.

The equation of the variation in time of the $\dot{Z}(\tau)$ phase change front position is:

$$\frac{\mathrm{d}\dot{Z}(\mathrm{Fo}_F)}{\mathrm{d}\,\mathrm{Fo}_F} \cong \left.\frac{\partial \Theta_0}{\partial \dot{y}}\right|_{\dot{y}=\dot{Z}} + \beta \cdot \left.\frac{\partial \Theta_1}{\partial \dot{y}}\right|_{\dot{y}=\dot{Z}} \tag{13}$$

where $\mathrm{Fo}_F = \frac{a_L \cdot \tau}{\Delta^2} \cdot \frac{c_L}{L} \cdot (t_0 - t_F)$ is the Fourier number for phase change processes and:

$$\Theta_0(\dot{Z}, \dot{y}) = \frac{1 + \mathrm{Bi}_F \cdot (1 - \dot{y}) + Bi_F \cdot \frac{t_0 - t}{t_0 - t_F} \cdot (\dot{y} - \dot{Z})}{1 + Bi_i \cdot (1 - \dot{Z})} \tag{14.1}$$

$$\Theta_1(\dot{Z}, \dot{y}) = [a_1 \cdot (\mathrm{Bi}_F, \dot{Z}) \cdot \dot{y}^3 - a_2 \cdot (\mathrm{Bi}_F, \dot{Z}) \cdot \dot{y}^2 + a_3 \cdot (\mathrm{Bi}_F, \dot{Z}) \cdot \dot{y} - a_4 \cdot (\mathrm{Bi}_i, \dot{Z})] \cdot \left(1 - \frac{t_0 - t}{t_0 - t_F}\right)^2 \tag{14.2}$$

The combination of solutions (10), (14.1) and (14.2) with the relation defining the phase change front speed (13) generates a non-linear differential equation of the first rank, the solution of which is the composite function $\dot{Z}(\dot{x}, \mathrm{Fo}_F)$. The Runge-Kutta method of the 4th order is recommended for solving equation (13).

The solutions previously described allow the determination of the variation in time and space of the phase change front position and of the heat carrier temperature.

2. Mathematical model specific to the isothermal fluid and to the PCSU plane geometry

The model presented is a particular case of the previous one, by considering the isothermal environments adjoining storage units with fundamental elementary geometries, in this case plane geometry. The aims of the modeling are the same as those presented in the case of the variable temperature fluid, namely the assessment of the phase change front variation speed both in thermal

loading processes and in thermal unloading processes.

The solution similar to that specific to the variable temperature fluid is:

$$\Theta(\dot{Z},\dot{y}) = \sum_{j=0}^{n} \beta^{j} \cdot \Theta_{j}(\dot{Z},\dot{y})$$

The representative solution of the first order is:

$$\Theta(\dot{Z},\dot{y}) \cong \frac{1+\mathrm{Bi}_F\cdot(1-\dot{y})}{1+\mathrm{Bi}_F\cdot(1-\dot{Z})} + \beta\cdot\left\{\frac{\mathrm{Bi}_F^3}{6\cdot[1+\mathrm{Bi}_F\cdot(1-\dot{Z})]^3}\cdot\dot{y}^3 - \frac{(1+\mathrm{Bi}_F)\cdot\mathrm{Bi}_F^2}{2\cdot[1+\mathrm{Bi}_F\cdot(1-\dot{Z})]^2}\cdot\dot{y}^2 + C_1\cdot\dot{y} + C_2\right\} \quad (15)$$

where:

$$C_1 = \frac{2\cdot\mathrm{Bi}_F^2\cdot[3\cdot(1+\mathrm{Bi}_F)+\mathrm{Bi}_F^2] - \mathrm{Bi}_F^3\cdot\dot{Z}^2\cdot[3\cdot(1+\mathrm{Bi}_F)-\mathrm{Bi}_F\cdot\dot{Z}]}{6\cdot[1+\mathrm{Bi}_F\cdot(1-\dot{Z})]^4}$$

$$C_2 = -\frac{2\cdot\mathrm{Bi}_F^2\cdot\dot{Z}\cdot[3\cdot(1+\mathrm{Bi}_F)+\mathrm{Bi}_F^2] - \mathrm{Bi}_F^2\cdot\dot{Z}^2\cdot(1+\mathrm{Bi}_F)\cdot[3\cdot(1+\mathrm{Bi}_F)-\mathrm{Bi}_F\cdot\dot{Z}]}{6\cdot[1+\mathrm{Bi}_F\cdot(1-\dot{Z})]^4}$$

The phase change front variation speed is determined by a relation of the following form:

$$\frac{\mathrm{d}\dot{Z}}{\mathrm{d}\,\mathrm{Fo}_F} = \frac{\sum_{k=0}^{3} A_k\cdot\dot{Z}^k}{\sum_{k=0}^{4} B_k\cdot\dot{Z}^k} \quad (16)$$

where A_k and B_k are functions of β and Bi_F, and Fo_F is the Fourier dimensionless number for phase change processes. For the preliminary calculations used in testing the performance of different PCTSU, the zero rank approximation, namely the simplification β = 0 in relation (15) may be used with a conveniently sufficient accuracy. In this case, the use of relation (13) leads to a first rank differential equation with an analytical solution. We further exemplify this approach for cylindrical and spherical geometries.

3. Mathematical model specific to the isothermal fluid – cylindrical and spherical geometries

3.1. Infinite cylinder (with R radius)

$$\Theta \cong \frac{1-\mathrm{Bi}_F\cdot\ln\dot{r}}{1-\mathrm{Bi}_F\cdot\ln\dot{Z}} \quad (17)$$

$$\mathrm{Fo}_F \cong 0{,}25 + \frac{0{,}50}{\mathrm{Bi}_F} + \\ +0{,}50\cdot\dot{Z}^2\cdot\ln\ \dot{Z} - \left(0{,}25+\frac{0{,}50}{\mathrm{Bi}_F}\right)\cdot\dot{Z}^2 \quad (18)$$

$$\mathrm{Fo}_F\big|_{\dot{Z}=0} = 0{,}25 + \frac{0{,}50}{\mathrm{Bi}_F} \quad (19)$$

3.2. Sphere (with R radius)

$$\Theta \cong \frac{1-\mathrm{Bi}_F+\dfrac{\mathrm{Bi}_F}{\dot{r}}}{1-\mathrm{Bi}_F+\dfrac{\mathrm{Bi}_F}{\dot{Z}}} \quad (20)$$

$$\mathrm{Fo}_F \cong 0{,}167 + \frac{0{,}33}{\mathrm{Bi}_F} - 0{,}25\cdot\dot{Z}^2 + 0{,}34\cdot\frac{1-\mathrm{Bi}_F}{\mathrm{Bi}_F}\cdot\dot{Z}^3 \quad (21)$$

$$\mathrm{Fo}_F\big|_{\dot{Z}=0} = 0{,}167 + \frac{0{,}33}{\mathrm{Bi}_F} \quad (22)$$

with: $\dot{r} = \dfrac{r}{R};\ \dot{Z} = \dfrac{Z}{R};$

$$\mathrm{Bi}_F = \frac{\alpha_F R}{\lambda};\ \mathrm{Fo}_F = \frac{a\tau}{R^2}\cdot\beta\,.$$

EXPERIMENTAL VALIDATION

The experiments use two substances representative in terms of thermodynamic characteristics, namely Glauber salt ($Na_2SO_4\cdot 10H_2O$), prepared according to a recipe minimizing the sub-cooling and phases separation risk (L = 164 kJ / kg, t_F = 32.5°C) and maleic anhydride ($C_4H_2O_3$) (L = 179 kJ / kg, t_F = 52°C). Even if these substances may raise risks in practical application, their use in experiments of validating PCTSU dimensioning mathematical models allows a proper quantification of the thermodynamic parameters by stability and recurrence.

- Cylindrical storage units ($2R$ = 0,05 m, h = 0,32 m) and substance used – maleic anhydride (Fig. 2). The temperature sensors (thermocouples) fixed inside the cylinder are numbered from 1 to 5.

- Thermal *loading* (Fig. 3) was performed with water as the heat carrier (ϑ_c = 62,5°C for the cylinder horizontal position): experimentally, the whole loading time-lag is of 2.25 h and according to the mathematical model it is of 2.37 h (relation (18) for $\dot{Z}=1$); the error is of 5.34 % which

confirms the accuracy of the mathematical model. The completion of the thermal loading process coincides with the moment when the temperature indicated by thermocouple no. 5 exceeds value t_F = 52°C. The diagram in fig. 3 shows that thermocouples 1 and 2 fixed, the same as thermocouple 5, in the cylinder central axis indicates the transition to the liquid phase of the substance about 1.6 hours before thermocouple 5. The thermal flow in the zones where thermocouples 1 and 2 are located (close to the cylinder ends) exceeds the thermal flow in the zone where thermocouple 5 is placed, which entails the modification of the phase change front speed, meaning that the latter increases.

The same experiment in the case of the vertical cylinder generates significant deviations in estimating the thermal framing compared to the measurements result (1.54 h). The difference results from the generation convective currents in the liquid phase volume. The currents inside the liquid phase intensify the heat transfer towards the solid phase, on one hand and interfere mechanically by eroding the inter-phase boundary, considerably increasing in time the heat transfer surface compared to the theoretical value, on the other hand.

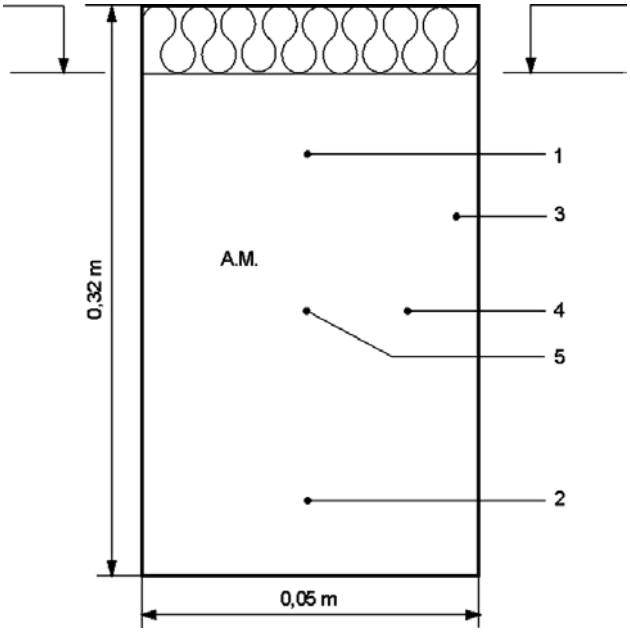

Fig. 2 – Maleic anhydride cylindrical storage unit and temperature sensors inside the cylinder.

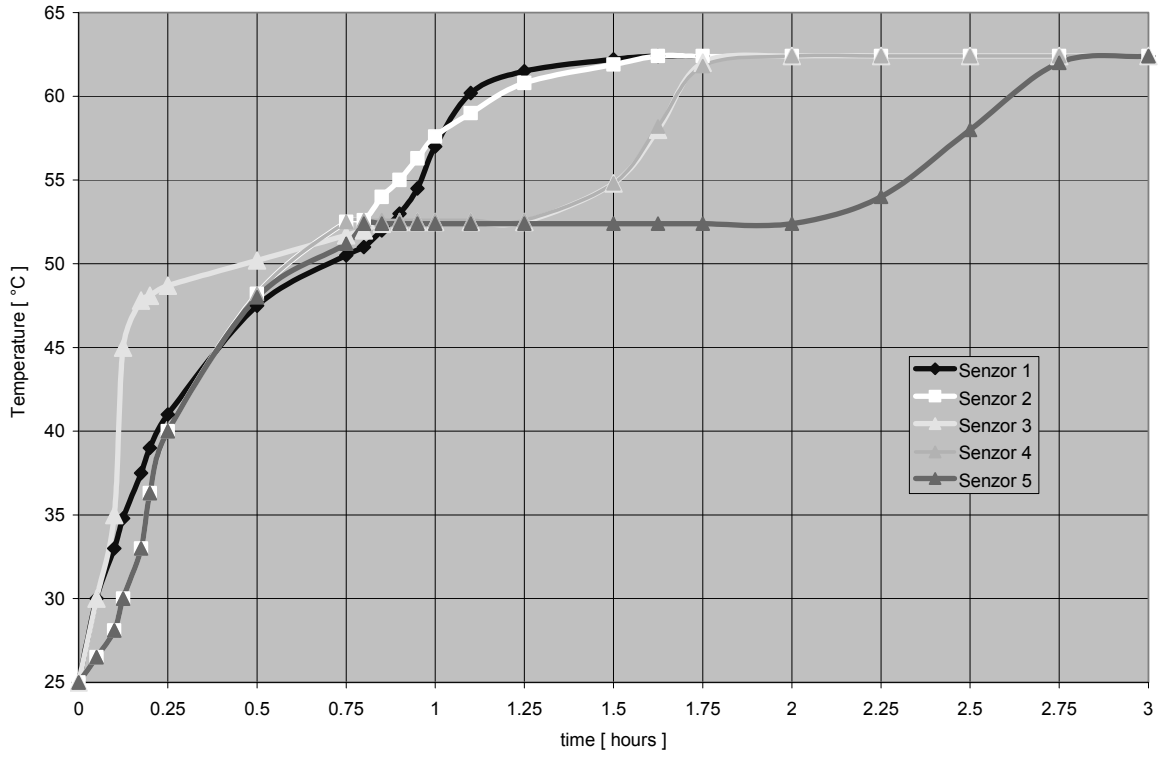

Fig. 3 – Temperature time variation of maleic anhidride storage unit – horizontal cylinder – 62.5°C water heating.

Thermal unloading (Fig. 4) was performed with water as heat carrier (ϑ_R = 24°C for the cylinder horizontal position): experimentally, the whole unloading time-lag is of 1.50 h and according to the mathematical model it is of 1.56 h; the error is of 4 % which confirms the mathematical model prediction. The experiment on the vertical cylinder observes the validation range specific to the horizontal position as a result of a solidification characteristic of the maleic anhydride by generating a reticular network of solidified threads forming solidification nuclei on one hand and annuls the effect of natural convection inside the liquid phase on the other hand. The reticular network formed within the cylinder influences the phase change process but in fact the thermal unloading decision is made simultaneously with the indications of thermocouples 5 and 4.

Sphere type storage units ($2R$ = 0,04 m) and substance used – Glauber salt – the PCSU is made of 7,000 spheres containing 350 kg of Glauber salt, air flow-rate = 1,130 m^3 / h, thermal capacity = 65 MJ of which 88 % represents the latent heat (Fig. 5). PCTSU is thermally insulated on its side so that it may be considered adiabatic. The air inlet in PCTSU is performed by means of an air pressure uniformity security chamber (CL) the main function of which is to provide a uniform speed profile in the storage unit cross-section, which provides a maximum efficiency to the heat transfer process between the heat carrier (air) and the spherical components.

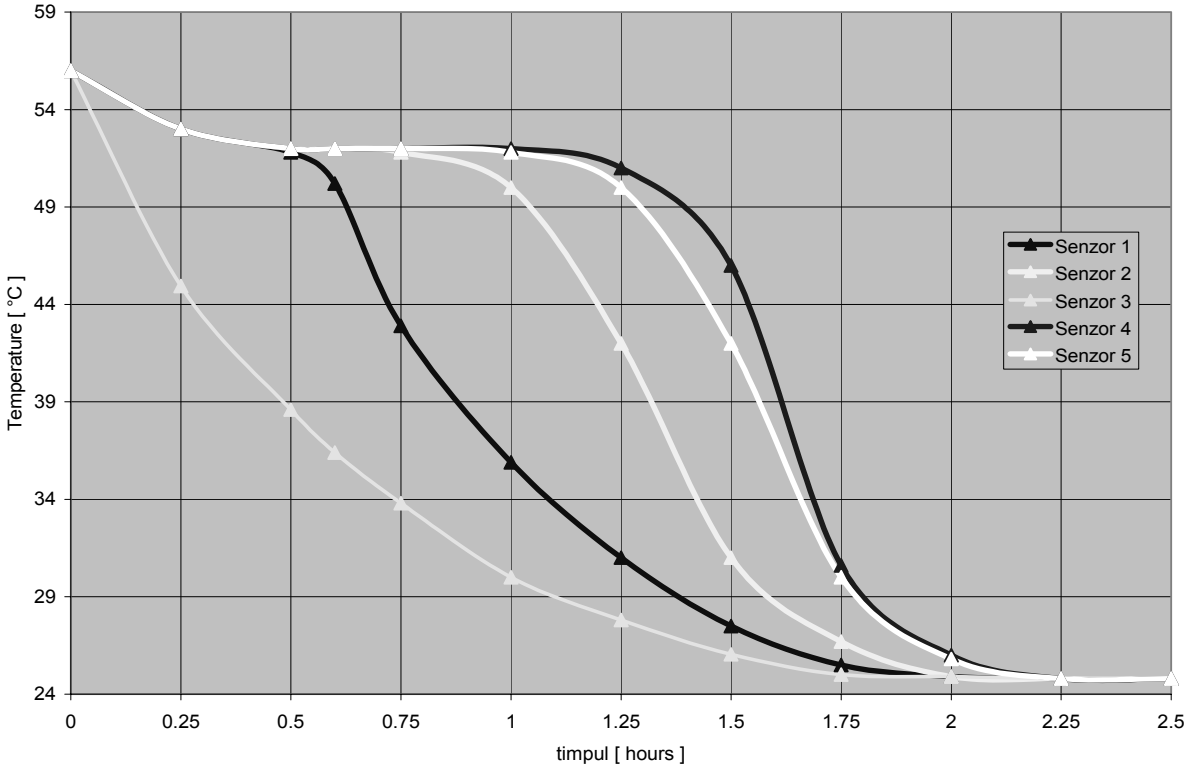

Fig. 4 – Temperature time variation of maleic anhidride storage unit – 24°C water cooling.

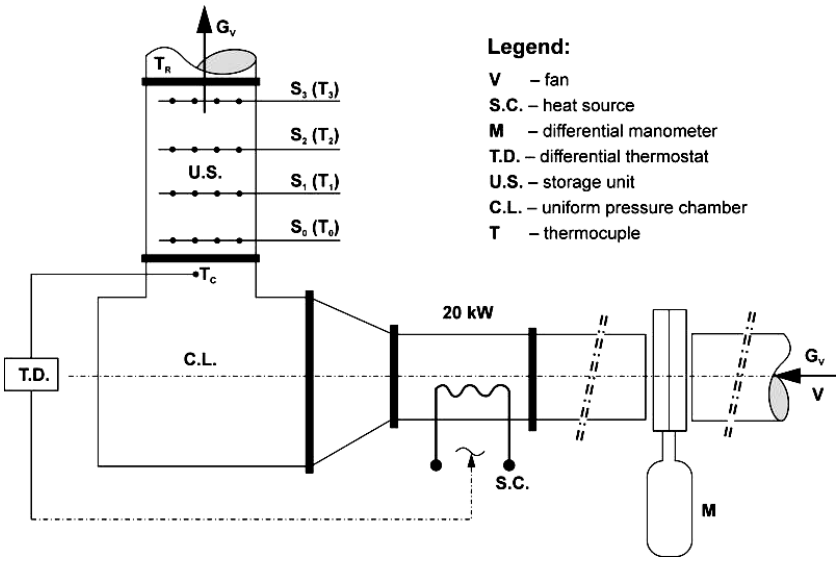

Fig. 5 – Glauber salt sphere type storage units.

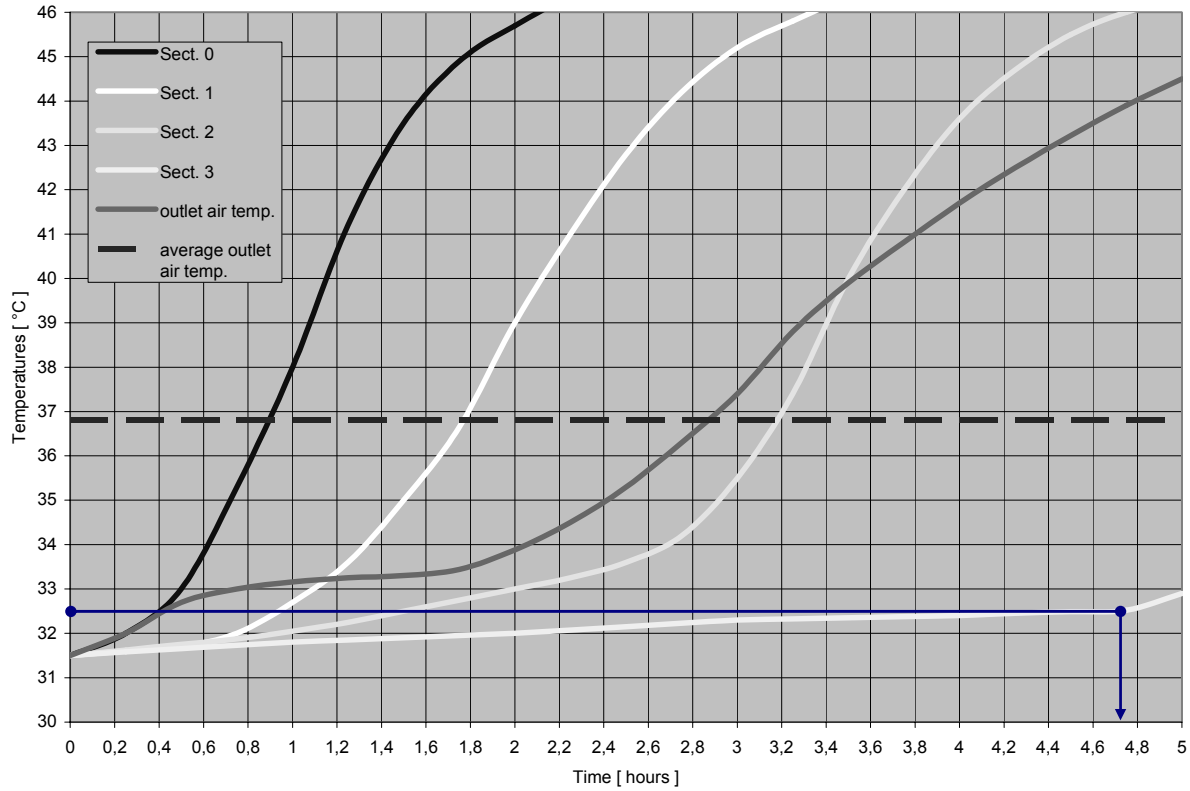

Fig. 6 – Thermal loading of the Glauber salt storage unit – 65 MJ heat capacity – 47°C air heat carrier.

The thermal loading was performed with air as heat carrier (ϑ_c = 47°C). Experimentally results are presented Fig. 6.

The completion of the PCTSU thermal loading is signaled by the indication that the average temperature of the substance in the spheres in section S_3 increases over t_F = 32.5°C shown by the thermocouples fixed in the middle of the spheres forming the PCTSU. The whole loading time-lag is of 4.75 h (Fig. 6). The heat carrier thermal balance leads to the following equation:

$$\bar{t}_1(\tau_f) = t_0 - \frac{47.7}{\tau_f} \tag{23}$$

where $\bar{t}_1(\tau_f)$ represents the air temperature average value in the PCTSU outlet section, in the time-lag $\tau \in [0, \tau_f]$ and t_0 is the constant temperature of the air inlet in the PCTSU.

Moreover, following the heat transfer process dynamics in the spheres forming the PCTSU, the average temperature in the air outlet section, determined by calculation for the same time-lag, based on a model similar to that presented in item 1 of this report, where the condition of completing the phase change process ($\dot{Z} = 1$) is imposed, leads to the following relation:

$$v(\tau_f) = 22.85 \cdot \frac{1 - \left[1 + \frac{13.85}{\sqrt{v(\tau_f)}}\right]^{-2}}{\tau_f} \tag{24}$$

where $v = \bar{t}_2(\tau_f) - t_F$.

The equality of values $\bar{t}_1(\tau_f)$ and $\bar{t}_2(\tau_f)$ implies the equality of relations (24) and (25) which generates a non-linear algebraic equation the solution of which is τ_f = 4.9 h. The error is of 3.16 % in terms of the value provided by the experiment, τ_f = 4.75 h which confirms the accuracy of the mathematical model.

CONCLUSIONS

This report presents the analytical solutions of Stefan problem based on the modified perturbed factor method, specific to storage units with elementary geometry: plane, cylindrical and spherical. The model includes simplifications based on the fact that the thermal energy accumulated by the internal energy modification process represents an insignificant rate of the whole quantity of accumulated thermal energy. This hypothesis is supported by the experimental validation focused on the heat storage units elementary models as well as on full-scale complete heat storage units. The energy performance assessment errors range between 3.16 % and 5.64 %, which represents an acceptable validation of the calculation models. Based on the mathematical models worked out and

experimentally validated, a decision analysis was performed on the necessary characteristics of the materials contained by PCTSU with reference to the thermal processes specific to buildings. For instance, as concerns the supply of domestic hot water, the easiest application of PCTSU by combination with the use of solar energy in the hot season emphasizes the necessity of using phase change materials with a phase change latent heat of at least so as to obtain a significant reduction of the thermal storage units volumes (Fig. 7).

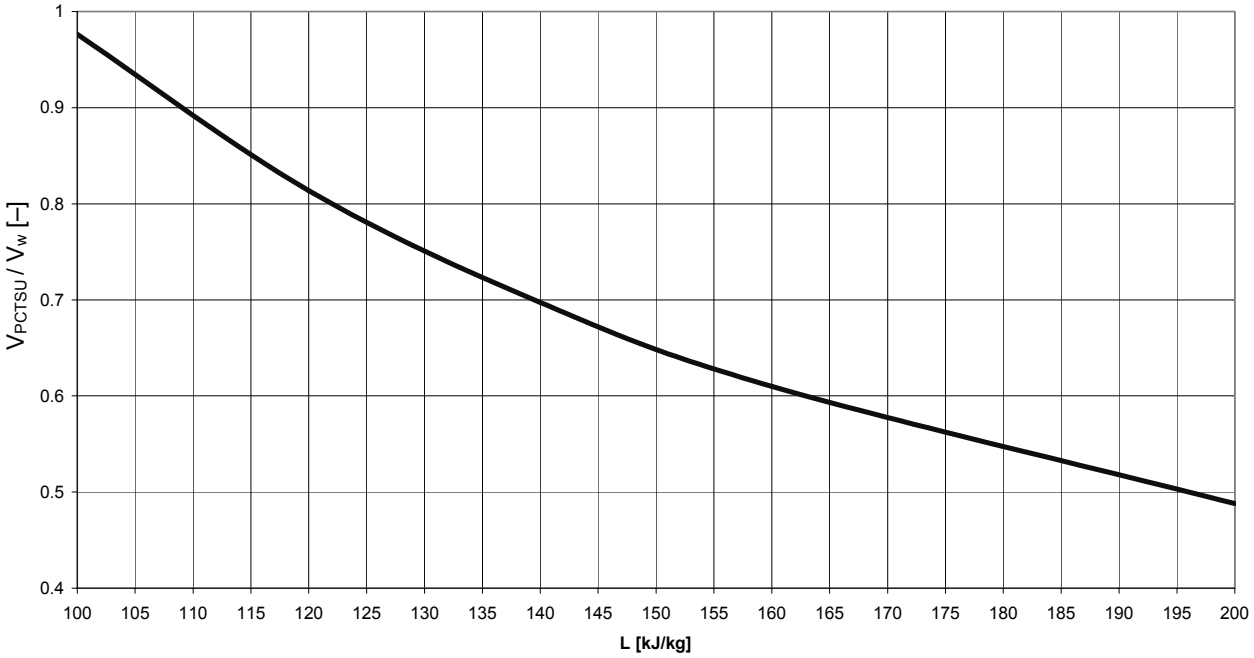

Fig. 7 – Ratio between PCTSU volume and water storage units according to PCM latent heat.

The calculation models presented in this report may be used both in defining the field of use of different phase change materials and in dimensioning the PCTSU forming the thermal systems specific to buildings with high energy performances, belonging to the category of buildings with an energy consumption close to zero.

NOMENCLATURE

Z – space coordinate indicating the phase change front position in terms of the boundary adjoining the heat carrier trace [m];
t – heat carrier temperature [°C];
ϑ – phase change material temperature [°C];
τ – time [s, h];
τ_f – process duration since moment $\tau = 0$ [s, h];
G – heat carrier mass flow-rate [kg /m^{2}s]
c – heat carrier specific heat [J / kg K];
c_L – mass specific heat of the liquid phase [J / kg K];
λ_L – liquid phase thermal conductivity [W / m K];
ρ_L – liquid phase density [kg / m^3];
a_L – liquid phase thermal diffusivity [m^2/s];
L – phase change latent heat [kJ / kg];
α_F – convection heat transfer coefficient corresponding to the heat transfer between the heat carrier and the phase change material [W / m^{2}K];

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