ACADEMIA ROMÂNĂ
Revue Roumaine de Chimie
http://web.icf.ro/rrch/

Rev. Roum. Chim.,
2021, *66*(7), 593–603
DOI: 10.33224/rrch.2021.66.7.02

Papers

COMPUTING TOPOLOGICAL INDICES FOR SILICATES AND HONEYCOMB NETWORKS

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Received October 3, 2018

$$F(G) = \sum_{v \in V(G)} d(v)^3 = \sum_{uv \in E(G)} (du^2 + dv^2)$$

A chemical graph is hydrogen depleted chemical structurei n which vertices denote atoms and edges denote the bonds. There are certain types of topological indices like distance based, degree based and counting related topological indices. In this article we gave expatiations for forgotten topological index, first and second Zagreb index, hyper Zagreb index, multiplicative Zagreb indices ass well as first and second Zagreb polynomials for some families of chemical networks.

INTRODUCTION

In mathematical chemistry, we discuss and predict some important properties of a chemical structure by using mathematical techniques. Chemical graph theory is a branch of mathematical Chemistry in which we apply tools from graph theory to mathematically model the chemical phenomenon. This theory plays a noticeable role in the fields of chemical sciences. In last decade, graph theory has found a considerable used in this area of research. Graph theory has provided chemistry with a variety of useful tools, such as topological indices.

The nanostar dendrimers are part of a new group of macromolecules that appear to be photo funnel like artificial antennas. These macromolecules and more precisely those containing phosphorus are used in the formation of nanotubes, micro and macro capsules, nanolatex, colored glasses, chemical sensors, modified electrodes, etc.[1] Nanostar dendrimers are one of the main objective of nano biotechnology. They possess a well-defined molecular topology. Their step-wise growth follows a mathematical progression. Dendrimers are highly ordered branched macromolecules which have attracted much theoretical and experimental attention.

Let G be a simple graph, with set of vertices V and set of edges E. A molecular graph is a simple connected graph where vertices denote atoms and edges denote bonds between atoms of the chemical compound. A molecular descriptor is a single numerical value which correlates the chemical structure with certain physic chemical properties of the compounds and is invariant under graph automorphisms. A large class of molecular descriptors depend on degree of vertices and are called degree based molecular descriptors. Degree of a vertex, say, v is number of vertices joined to v

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by an edge of the graph, and denoted by (v). Followed by the first and second Zagreb indices, in 2015, F. Gutman introduced forgotten topological index (also called F-index) which was defined as:

$$F(G) = \sum_{v \in V(G)} d(v)^3 = \sum_{u,v \in E(G)} (du^2 + dv^2)$$

where d(v) is denoted as the degree of vertex v (the number of vertex adjacent to vertex v). Furtula and Gutman raised that the predictive ability of forgotten topological index is almost similar to that of first Zagreb index and for the acentric factor and entropy, and both of them obtain correlation coefficients larger than 0.95. This fact implies the reason why forgotten topological index is useful for testing the chemical and pharmacological properties of drug molecular structures. In 2014, Sunet *et al*. deduced some basic nature of forgotten topological index and reported that this index can reinforce the physico-chemical flexibility of Zagreb indices.

Let δ(G) and Δ(G) be the minimum and maximum degree of G, respectively. The edge set E(G) can be divided into several partitions: for any i and j, δ(G) ≤ i,j ≤Δ(G), let

$$E_{ij} = \{e = uv \in E(G) |\ d(v) = i, d(u) = j\}\ and\ n_{ij} = |E_{ij}|.$$

Zagreb indices are one of the oldest known topological invariants which first appeared as terms in a formula for analysis of π-electronenergy[5] and they grow with the branching of chemical graphs. Balaban *et al.*[2] named them “Zagreb group indices” which later on termed as first Zagreb index and second Zagreb index and are defined as:

$$M_1(G) = \sum_{uv \in E(G)} [deg(u) + deg(v)],$$

$$M_2(G) = \sum_{uv \in E(G)} [deg(u) \times deg(v)]$$

In 2013, Shirdel *et al.*[12] introduced hyper-Zagreb index which is defined as:

$$HM(G) = \sum_{uv \in E(G)} [deg(u) + deg(v)]^2$$

Ghorbani and Azimij defined first multiple Zagreb index $PM_1(G)$ and second multiple Zagreb index $PM_2(G)$ of a graphic G in 2012.[6] These are given by the following formulas:

$$PM_1(G) = \prod_{uv \in E(G)} [deg(u) + deg(v)],$$

$$PM_2(G) = \prod_{uv \in E(G)} [deg(u) \times deg(v)]$$

The first Zagreb polynomial $M_1(G,x)$ and second Zagreb polynomial $M_2(G,x)$ are defined as:

$$M_1(G,x) = \sum_{uv \in E(G)} x^{[deg(u)+deg(v)]},$$

$$M_2(G,x) = \sum_{uv \in E(G)} x^{[deg(u)+deg(v)]}$$

These new variants of Zagreb indices have been extensively studied recently.[1-13]

MAIN RESULTS

1) Silicate networks SL_n

Silicates the largest, very interesting and most complicated minerals by far. Silicates are obtained by using metal oxides or metal carbonates with sand, see Figure 1. A silicate network of dimension n symbolizes as SL_n, where n is the number of hexagons between the center and boundary of SL_n.

The edge partition of SL_n with respect to the degrees of the end-vertices of edges given by Table 1.

Table 1

(d_u,d_v)-type edge partition of SL_n

(d_u, d_v)	(3, 3)	(3, 6)	(6, 6)
No. of edges	6n	$18n^2 + 6n$	$18n^2 - 12n$

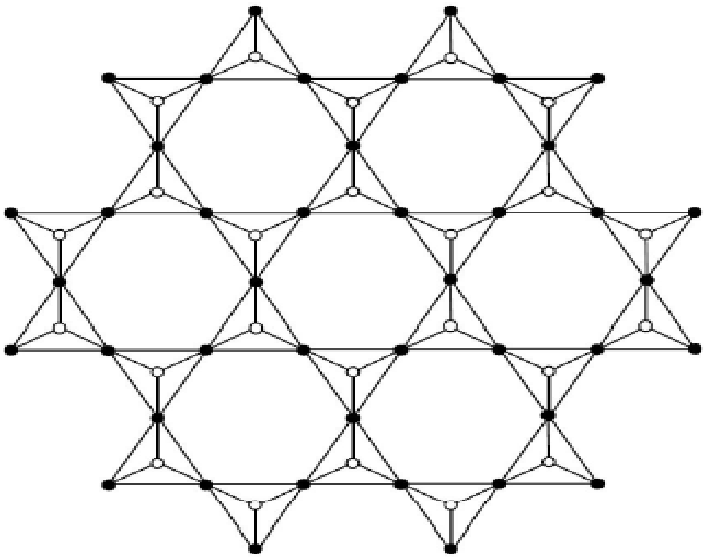

Fig. 1 – Silicate network of dimension two, the solid vertices denote the oxygen atoms whereas the plain vertices represent the silicon atoms.

Results for Silicates networks SL_n

Let G be the graph of Silicates networks SL_n.The edge set is partitioned into three sets, say, E_1, E_2, E_3 based on the degree of end vertices of each edge. E_1 contains $6n$ edges of type uv such that $\deg(u) = 3, \deg(v) = 3$, E_2 contains $18n^2 + 6n$ edges of type uv such that $\deg(u) = 3$, $\deg(v) = 6$, E_3 contains $18n^2 - 12n$ edges of type uv such that $\deg(u) = 6$, $\deg(v) = 6$. Now we compute our results as follows:

$$F(G) = \sum_{uv \in E(G)} du^2 + dv^2$$

$$F(SL_n) = \sum_{uv \in E_1} (du^2 + dv^2) + \sum_{uv \in E_2} (du^2 + dv^2) + \sum_{uv \in E_3} (du^2 + dv^2)$$

$$= 18|E_1(SL_n)| + 45\,|E_2(SL_n)| + 72|E_3(SL_n)|$$

$$= 18(6n) + 45(18n^2 + 6n) + 72(18n^2 - 12n) = 2106n^2 + 378n - 864$$

$$M_1(G) = \sum_{uv \in E(G)} [d_u + d_v]$$

$$M_1(SL_n) = \sum_{uv \in E_1} [d_u + d_v] + \sum_{uv \in E_2} [d_u + d_v] + \sum_{uv \in E_3} [d_u + d_v]$$

$$= 6\,|E_1(SL_n)| + 9\,|E_2(SL_n)| + 12\,|E_3(SL_n)|$$

$$= 6(6n) + 9(18n^2 + 6n) + 12(18n^2 - 12n) = 378n^2 - 54n$$

$$M_2(G) = \sum_{uv \in E(G)} [d_u \times d_v]$$

$$M_2(SL_n) = \sum_{uv \in E_1} [d_u \times d_v] + \sum_{uv \in E_2} [d_u \times d_v] + \sum_{uv \in E_3} [d_u \times d_v]$$

$$= 9\,|E_1(SL_n)| + 18\,|E_2(SL_n)| + 36|E_3(SL_n)|$$

$$= 9(6n) + 18(18n^2 + 6n) + 36(18n^2 - 12n) = 972n^2 - 270n$$

$$HM(G) = \sum_{uv\in E(G)} [d_u + d_v]^2$$

$$HM(SL_n) = \sum_{uv\in E_1} [d_u + d_v]^2 + \sum_{uv\in E_2} [d_u + d_v]^2 + + \sum_{uv\in E_3} [d_u + d_v]^2$$

$$= 36\,|E_1(SL_n)| + 81\,|E_2(SL_n)| + 144\,|E_3(SL_n)|$$

$$= 36(6n) + 81(18n^2 + 6n) + 144(18n^2 - 12n) = 4050n^2 - 1026n$$

$$PM_1(G) = \prod_{uv\in E(G)} [d_u + d_v]$$

$$PM_1(SL_n) = \prod_{uv\in E_1} [d_u + d_v] \times \prod_{uv\in E_2} [d_u + d_v] \times \prod_{uv\in E_3} [d_u + d_v]$$

$$= (6)^{|E_1(SL_n)|} \times (9)^{|E_2(SL_n)|} \times (12)^{|E_3(SL_n)|}$$

$$= (6)^{6n} \times (9)^{18n^2+6n} \times (12)^{18n^2-12n} = 2^{36n^2-18n} \times 3^{54n^2+6n}$$

$$PM_2(G) = \prod_{uv\in E(G)} [d_u \times d_v]$$

$$PM_2(SL_n) = \prod_{uv\in E_1} [d_u \times d_v] \times \prod_{uv\in E_2} [d_u \times d_v] \times \prod_{uv\in E_3} [d_u \times d_v]$$

$$= (9)^{|E_1(SL_n)|} \times (18)^{|E_2(SL_n)|} \times (36)^{|E_3(SL_n)|} = 2^{84n^2-18n} \times 3^{72n^2}$$

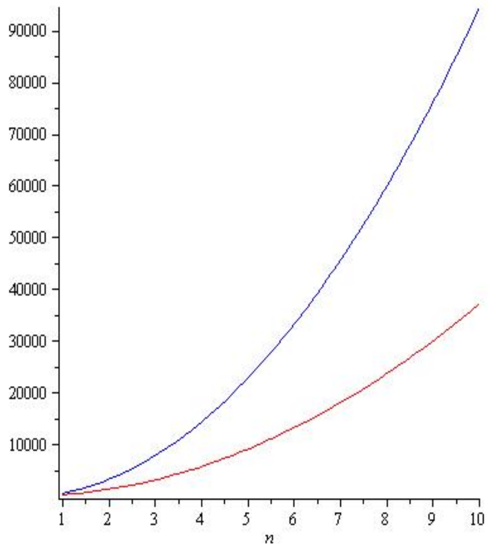

Fig. 2 – First and second Zagreb indices $M_1(G)$ and $M_2(G)$ of G equivalent to SL_n, Red and Blue colors represent $M_1(G)$ and $M_2(G)$, respectively. We can see that, in the given domain, $M_2(G)$ is dominating than $M_1(G)$.

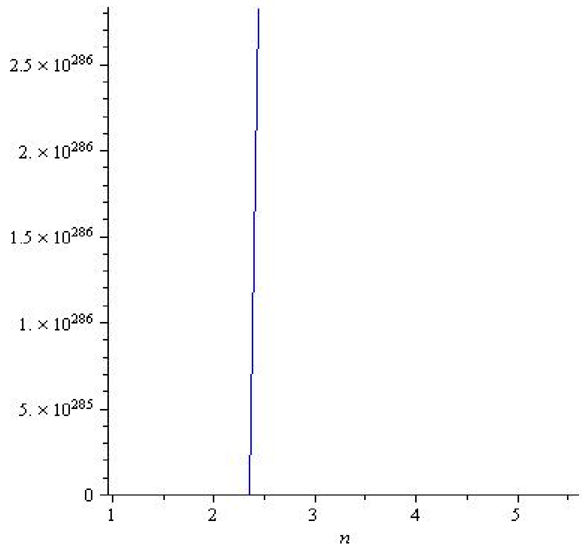

Fig. 3 – First and second multiple Zagreb indices $PM_1(G)$ and $PM_2(G)$ of G equivalent to SL_n, Red and Blue colors represent $PM_1(G)$ and $PM_2(G)$, respectively. We can see that, in the given domain, both $PM_1(G)$ and $PM_2(G)$ are identical vertical lines.

$$M_1(G,x) = \sum_{uv\in E(G)} x^{[d_u+d_v]}$$

$$M_1(SL_n,x) = \sum_{uv\in E_1} x^{[d_u+d_v]} + \sum_{uv\in E_2} x^{[d_u+d_v]} + \sum_{uv\in E_3} x^{[d_u+d_v]}$$

$$M_1(SL_n,x) = (|E_1(SL_n)|)x^6 + (|E_2(SL_n)|)x^9 + (|E_3(SL_n)|)x^{12}$$
$$= (6n)x^6 + (18n^2+6)x^9 + (18n^2-12n)x^{12}$$
$$M_2(G,x) = \sum_{uv\in E(G)} x^{[d_u\times d_v]}$$
$$M_2(SL_n,x) = \sum_{uv\in E_1} x^{[d_u\times d_v]} + \sum_{uv\in E_2} x^{[d_u\times d_v]} + \sum_{uv\in E_3} x^{[d_u\times d_v]}$$
$$M_2(SL_n,x) = (|E_1(SL_n)|)x^9 + (|E_2(SL_n)|)x^{18} + (|E_3(SL_n)|)x^{36}$$
$$= (6n)x^9 + (18n^2+6)x^{18} + (18n^2-12n)x^{36}$$

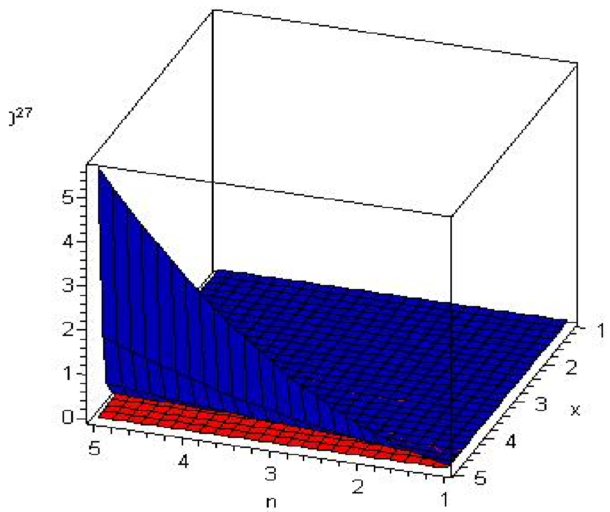

Fig. 4 – First and second Zagreb polynomial $M_1(G, x)$ and $M_2(G, x)$ of G equivalent to SL_n, Red and Blue colors represent $M_1(G, x)$ and $M_2(G, x)$, respectively. We can see that, in the given domain, $M_2(G, x)$ is more dominating than $M_1(G, x)$.

2) Chain Silicate Networks CS_n

Next, we consider another family of silicate networks named as chain silicate networks and then compute its certain degree based topological indices. Here we provide chain silicate networks CS_n of dimension n as follows: a chain silicate a network of dimension n symbolizes as CS_n is obtained by arranging n tetrahedral linearly, see Figure 5.

The number I of vertices in CS_n is 3n + 1 and number of edges are 6n.

Let $G = (V, E)$ be a graph of chain silicate networks, edge partition based ion degrees of end vertices of each edge is as.

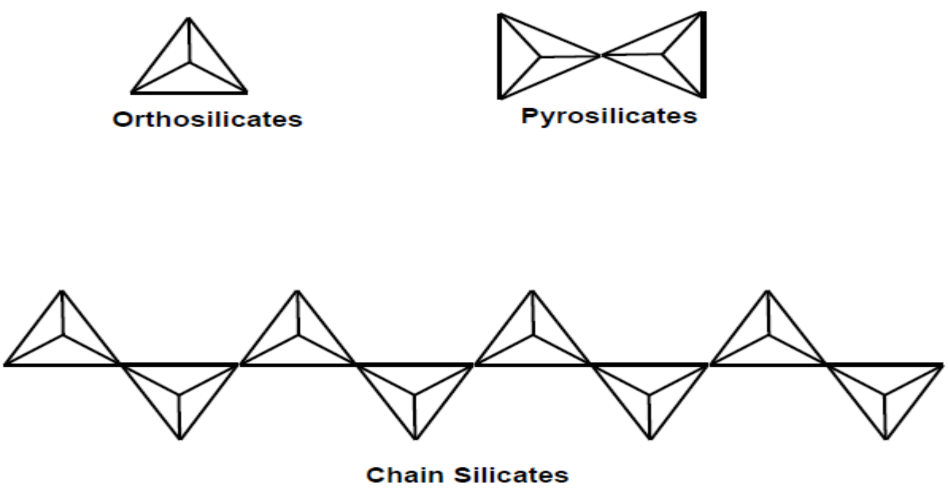

Fig. 5 – Ortho, Pyro and chain silicates.

Table 2
(d_u, d_v)-type edge partition of CS_n

(d_u, d_v)	(3, 3)	(3,6)	(6, 6)
No. of edges n=1	6	0	0
$n \geq 2$	n+4	4n-2	n-2

Results for Chain Silicate Networks CS_n

Let G be the graph of Chain Silicates networks CS_n. The edge set is partitioned into three sets, say, E_1, E_2, E_3 based on the degree of end vertices of each edge. E_1 contains 6 edges for n=1 and contain n+4 edges for $n \geq 2$, of type uv such that $\deg(u) = 3, \deg(v) = 3$, E_2 contains $4n-2$ edges for $n \geq 2$, of type uv such that $\deg(u) = 3$, $\deg(v) = 6$. E_3 contains n 2 edges for $n \geq 2$, of type uv such that $\deg(u) = 6$, $\deg(v) = 6$.

$$F(G) = \sum_{uv \in E(G)} du^2 + dv^2$$

$$F(CS_n) = \sum_{uv \in E_1} (du^2 + dv^2) + \sum_{uv \in E_1} (du^2 + dv^2) + \sum_{uv \in E_1} (du^2 + dv^2)$$

$$= 18|E_1(CS_n)| = 18(6) = 108 \qquad for\ n = 1$$

$$= 18|E_1(CS_n)| + 45\ |E_2(CS_n)| + 72|E_3(CS_n)| \qquad for\ n \geq 2$$

$$= 18(n+4) + 45(4n-2) + 72(n-2) = 270n - 162$$

$$M_1(G) = \sum_{uv \in E(G)} [d_u + d_v]$$

$$M_1(CS_n) = \sum_{uv \in E_1} [d_u + d_v] + \sum_{uv \in E_2} [d_u + d_v] + \sum_{uv \in E_3} [d_u + d_v]$$

$$= 6\ |E_1(SL_n)| = 6(6) = 36 \qquad for\ n = 1$$

$$= 6\ |E_1(SL_n)| + 9\ |E_2(SL_n)| + 12\ |E_3(SL_n)| \qquad for\ n \geq 2$$

$$= 6(n+4) + 9(4n-2) + 12(n-2) = 54n - 18$$

$$M_2(G) = \sum_{uv \in E(G)} [d_u \times d_v]$$

$$M_2(CS_n) = \sum_{uv \in E_1} [d_u \times d_v] + \sum_{uv \in E_2} [d_u \times d_v] + \sum_{uv \in E_3} [d_u \times d_v]$$

$$= 9\ |E_1(CS_n)| = 9(6) = 54 \qquad for\ n = 1$$

$$= 9\ |E_1(CS_n)| + 18\ |E_2(CS_n)| + 36|E_3(CS_n)| \qquad for\ n \geq 2$$

$$= 9(n+4) + 18(4n-2) + 36(n-2) = 117n - 72$$

$$HM(G) = \sum_{uv \in E(G)} [d_u + d_v]^2$$

$$HM(CS_n) = \sum_{uv \in E_1} [d_u + d_v]^2 + \sum_{uv \in E_2} [d_u + d_v]^2 + + \sum_{uv \in E_3} [d_u + d_v]^2$$

$$= 36\ |E_1(CS_n)| = 36(6) = 216 \quad for\ n = 1$$

$$= 36\ |E_1(CS_n)| + 81\ |E_2(CS_n)| + 144\ |E_3(CS_n)| \quad for\ n \geq 2$$

$$= 36(n+4) + 81(4n-2) + 144(n-2) = 504n - 306$$

$$PM_1(G) = \prod_{uv \in E(G)} [d_u + d_v]$$

$$PM_1(CS_n) = \prod_{uv \in E_1} [d_u + d_v] \times \prod_{uv \in E_2} [d_u + d_v] \times \prod_{uv \in E_3} [d_u + d_v]$$

$$= (6)^{|E_1(CS_n)|} = 6^6 \qquad for\ n = 1$$

$$= (6)^{|E_1(CS_n)|} \times (9)^{|E_2(CS_n)|} \times (12)^{|E_3(CS_n)|} \quad for\ n \geq 2$$
$$= (6)^{n+4} \times (9)^{4n-2} \times (12)^{n-2} = 2^{3n} \times 3^{9n-6}$$
$$PM_2(G) = \prod_{uv\in E(G)} [d_u \times d_v]$$
$$PM_2(CS_n) = \prod_{uv\in E_1} [d_u \times d_v] \times \prod_{uv\in E_2} [d_u \times d_v] \times \prod_{uv\in E_3} [d_u \times d_v]$$
$$= (9)^{|E_1(CS_n)|} = 9^6 \quad for\ n = 1$$
$$= (9)^{|E_1(CS_n)|} \times (18)^{|E_2(CS_n)|} \times (36)^{|E_3(CS_n)|} \quad for\ n \geq 2$$
$$= (9)^{n+4} \times (18)^{4n-2} \times (36)^{n-2} = 2^{6n-6} \times 3^{12n}$$

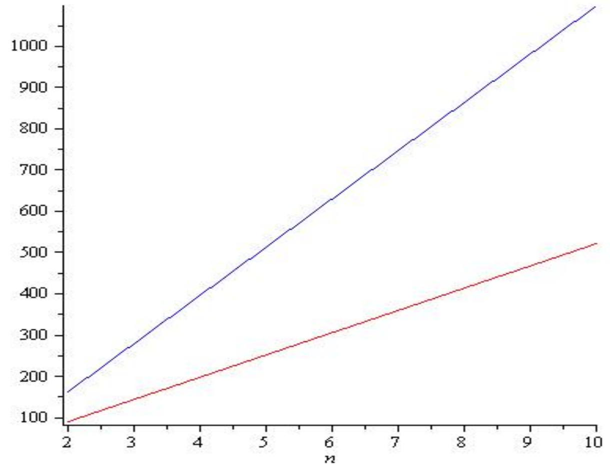

Fig. 6 – First and second Zagreb indices $M_1(G)$ and $M_2(G)$ of G equivalent to CS_n , Red and Blue colors represent $M_1(G)$ and $M_2(G)$, respectively. We can see that, in the given domain, $M_1(G)$, $M_2(G)$ are equally distributed.

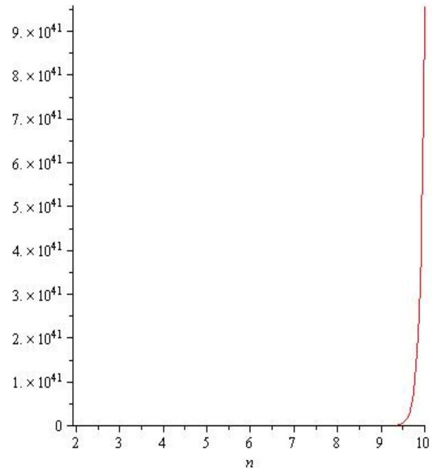

Fig. 7 – First and second multiple Zagreb indices $PM_1(G)$ and $PM_2(G)$ of G equivalent to CS_n , Red and Blue colors represent $PM_1(G)$ and $PM_2(G)$, respectively. We can see that, in the given domain, $PM_2(G)$ and $PM_1(G)$ are identical curve.

First and second Zagreb polynomial of CS_n are computed as:

$$M_1(G,x) = \sum_{uv\in E(G)} x^{[d_u+d_v]}$$
$$M_1(CS_n,x) = \sum_{uv\in E_1} x^{[d_u+d_v]} + \sum_{uv\in E_2} x^{[d_u+d_v]} + \sum_{uv\in E_3} x^{[d_u+d_v]}$$
$$= (|E_1(CS_n)|)x^6 = 6x^6 \quad for\ n = 1$$
$$M_1(CS_n,x) = (|E_1(CS_n)|)x^6 + (|E_2(CS_n)|)x^9 + (|E_3(CS_n)|)x^{12} \quad for\ n \geq 2$$
$$= (n+4)x^6 + (4n-2)x^9 + (n-2)x^{12}$$
$$M_2(G,x) = \sum_{uv\in E(G)} x^{[d_u\times d_v]}$$
$$M_2(CS_n,x) = \sum_{uv\in E_1} x^{[d_u\times d_v]} + \sum_{uv\in E_2} x^{[d_u\times d_v]} + \sum_{uv\in E_3} x^{[d_u\times d_v]}$$
$$= (|E_1(CS_n)|)x^9 = 6x^9 \quad for\ n \geq 1$$
$$M_2(CS_n,x) = (|E_1(CS_n)|)x^9 + (|E_2(CS_n)|)x^{18} + (|E_3(CS_n)|)x^{36} \quad for\ n \geq 2$$
$$= (n+4)x^9 + (4n-2)x^{18} + (n-2)x^{36}$$

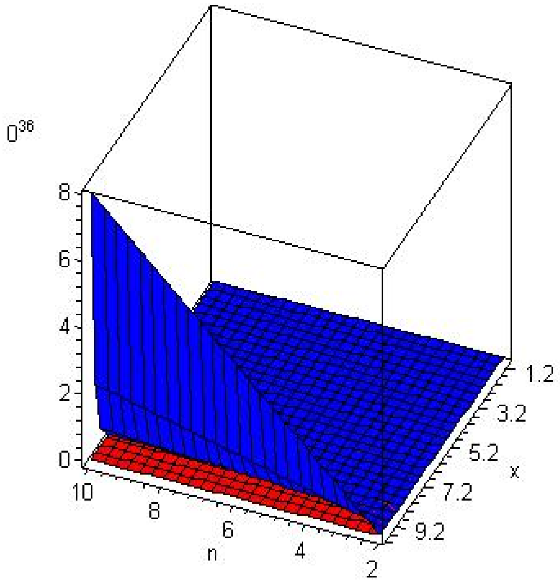

Fig. 8 – First and second Zagreb polynomial $M_1(G, x)$ and $M_2(G)$ of G equivalent to CS_n, Red and Blue colors represent $M_1(G, x)$ and $M_2(G, x)$, respectively. We can see that, in the given domain, $M_2(G, x)$ is more dominating than $M_1(G, x)$

3) Hexagonal Network HX_n

It is known that there exist three irregular plane tiling's with composition of same kind of regular polygons such as triangular, hexagonal and square. In the construction of hexagonal networks, triangular tiling is used, see Figure 9. A hexagonal network of dimension n is usually denoted as HX_n, where n is the number of vertices on each side of hexagon. The number iof vertices in hexagonal networks HX_n iare $3n^2 - 3n + 1$ and number of edges are $9n^2 - 15n + 6$.

The edge partition of HX_n with respect to the degrees of the in-services of edges given by Table 3.

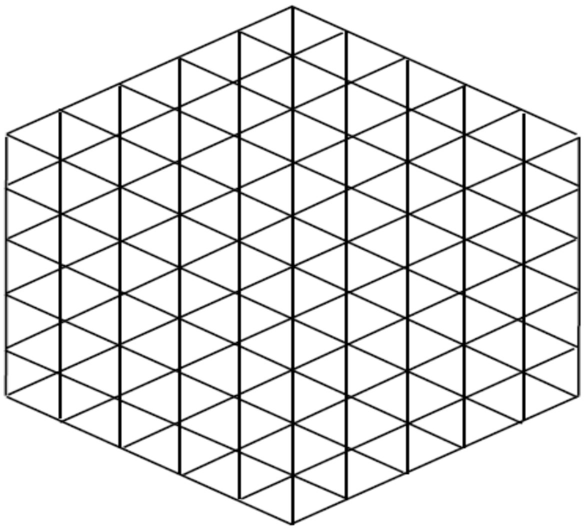

Fig. 9 – Hexagonal network of dimension 6.

Table 3

(d_u, d_v)-type edge partition of HX_n.

(d_u, d_v)	(3, 4)	(3, 6)	(4, 4)	(4, 6)	(6, 6)
No. of edges	12	6	$6n - 18$	$12n - 24$	$9n^2 - 33n + 30$

Results for Hexagonal Network HX_n

Let G be the graph of hexagonal networks, HX_n. The edge set is partitioned into five sets, say, E_1, E_2, E_3, E_4, E_5 based on the degree of end vertices of each edge E_1 contains 12 edges of type uv such that $\deg(u) = 3, \deg(v) = 4$, E_2 contains 6 edges of type uv such that $\deg(u) = 3$, $\deg(v) = 6$,E_3 contains $6n - 18$ edges of type uv such that $\deg(u) = \deg(v) = 4$, E_4 contains $12n \quad 24$ edges of type uv such that deg(u) = 4, deg (v) = 6, E_5 contains $9n^2 - 33 + 30$ edges of type uv such that deg(u) = 6, deg(v)=6.

$$F(G) = \sum_{uv\in E(G)} du^2 + dv^2$$

$$F(HX_n) = \sum_{uv\in E_1} du^2 + dv^2 + \sum_{uv\in E_2} du^2 + dv^2 + \sum_{uv\in E_3} du^2 + dv^2 + \sum_{uv\in E_4} du^2 + dv^2$$
$$+ \sum\nolimits_{uv\in E_5} du^2 + dv^2$$
$$= 25|E_1(HX_n)| + 45\,|E_2(HX_n)| + 32|E_3(HX_n)| + 52|E_4(HX_n)| + 72|E_5(HX_n)|$$
$$- 25(12) + 45\,(6) + 32(6n - 18) + 52(12n - 24) + 12(9n^2 - 33n + 30)$$
$$= 648n^2 - 2376n + 2160$$

$$M_1(G) = \sum\nolimits_{uv\in E(G)}[d_u + d_v]$$
$$M_1(HX_n) = \sum\nolimits_{uv\in E_1}[d_u + d_v] + \sum\nolimits_{uv\in E_2}[d_u + d_v] + \sum\nolimits_{uv\in E_3}[d_u + d_v] + \sum\nolimits_{uv\in E_4}[d_u + d_v]$$
$$+ \sum\nolimits_{uv\in E_5}[d_u + d_v]$$
$$= 7|E_1(HX_n)| + 9\,|E_2(HX_n)| + 8|E_3(HX_n)| + 10|E_4(HX_n)| + 12|E_5(HX_n)|$$
$$= 7(12) + 9(6) + 8(6n - 18) + 10(12n - 24) + 12(9n^2 - 33n + 30)$$
$$= 108n^2 - 228n - 606$$

$$M_2(G) = \sum_{uv\in E(G)} [d_u \times d_v]$$
$$M_2(HX_n) = \sum\nolimits_{uv\in E_1}[d_u \times d_v] + \sum\nolimits_{uv\in E_2}[d_u \times d_v] + \sum\nolimits_{uv\in E_3}[d_u \times d_v] + \sum\nolimits_{uv\in E_4}[d_u \times d_v]$$
$$+ \sum\nolimits_{uv\in E_5}[d_u \times d_v]$$
$$= 12\,|E_1(HX_n)| + 18\,|E_2(HX_n)| + 16|E_3(HX_n)| + 24|E_4(HX_n)| + 36|E_5(HX_n)|$$
$$= 12(12) + 18(6) + 16(6n - 18) + 24(12n - 24) + 36(9n^2 - 33n + 30)$$
$$= 324n^2 - 804n + 468$$

$$HM(G) = \sum_{uv\in E(G)} [d_u + d_v]^2$$
$$HM(HX_n) = \sum_{uv\in E_1} [d_u + d_v]^2 + \sum_{uv\in E_2} [d_u + d_v]^2 + \sum_{uv\in E_3} [d_u + d_v]^2 + \sum_{uv\in E_4} [d_u + d_v]^2$$
$$+ \sum_{uv\in E_5} [d_u + d_v]^2$$
$$= 49|E_1(HX_n)| + 81\,|E_2(HX_n)| + 64|E_3(HX_n)| + 100|E_4(HX_n)| + 144|E_5(HX_n)|$$
$$= 49(12) + 81(6) + 64(6n - 18) + 100(12n - 24) + 144(9n^2 - 33n + 30)$$
$$= 1296n^2 - 3168n + 1842$$

$$PM_1(G) = \prod\nolimits_{uv\in E(G)}[d_u + d_v]$$

$$PM_1(HX_n) = \prod_{uv\in E_1}[d_u + d_v] \times \prod_{uv\in E_2}[d_u + d_v] \times \prod_{uv\in E_3}[d_u + d_v] \times \prod_{uv\in E_4}[d_u + d_v] \times \prod_{uv\in E_5}[d_u + d_v]$$

$$= (7)^{|E_1(HX_n)|} \times (9)^{|E_2(HX_n)|} \times (8)^{|E_3(HX_n)|} \times (10)^{|E_4(HX_n)|} \times (12)^{|E_5(HX_n)|}$$

$$= (7)^{12} \times (9)^{6} \times (8)^{6n-18} \times (10)^{12n-24} \times (12)^{9n^2-33n+30}$$

$$= 2^{18n^2-36n-18} \times 3^{9n^2-33n+42} \times 5^{12n-24} \times 7^{12}$$

$$PM_2(G) = \prod_{uv\in E(G)}[d_u \times d_v]$$

$$PM_2(HX_n) = \prod_{uv\in E_1}[d_u \times d_v] \times \prod_{uv\in E_2}[d_u \times d_v] \times \prod_{uv\in E_3}[d_u \times d_v] \times \prod_{uv\in E_4}[d_u \times d_v] \times \prod\nolimits_{uv\in E_5}[d_u \times d_v]$$

$$= (12)^{|E_1(HX_n)|} \times (18)^{|E_2(HX_n)|} \times (16)^{|E_3(HX_n)|} \times (24)^{|E_4(HX_n)|} \times (36)^{|E_5(HX_n)|}$$

$$= (12)^{12} \times (18)^{6} \times (16)^{6n-18} \times (24)^{12n-24} \times 36^{9n^2-33n+30}$$

$$= 2^{18n^2-6n-54} \times 3^{18n^2-54n+60}$$

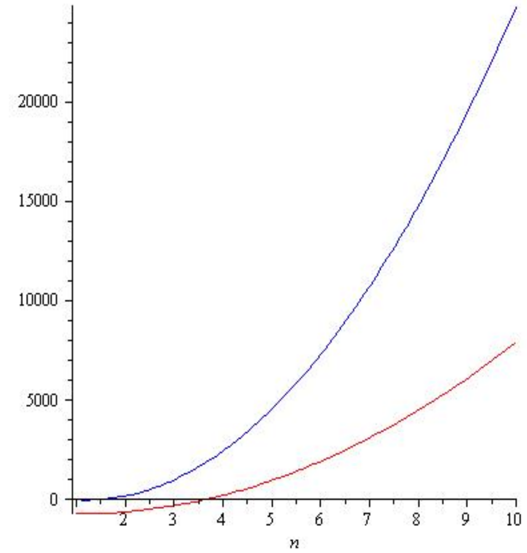

Fig. 10 – First and second Zagreb indices $M_1(G)$ and $M_2(G)$ of G equivalent to HX_n , Red and Blue colors represent $M_1(G)$ and $M_2(G)$, respectively. We can see that, in the given domain, $M_2(G)$ is more dominate than $M_1(G)$.

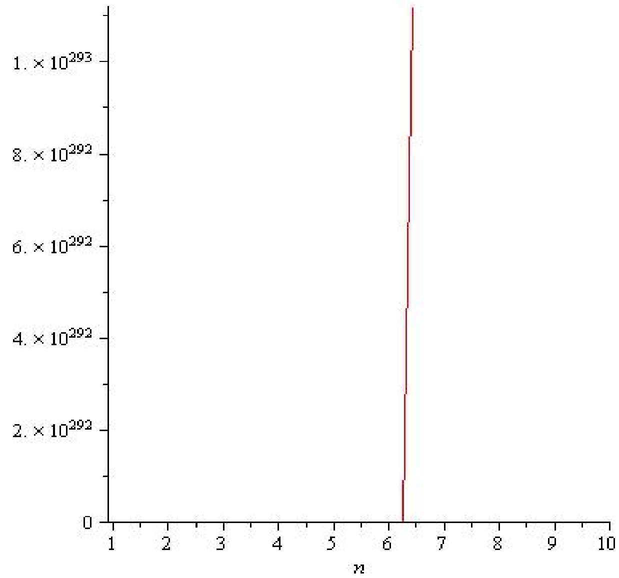

Fig. 11 – First and second multipleZagreb indices $PM_1(G)$ and $PM_2(G)$ of G equivalent to HX_n, Red and Blue colors represent $PM_1(G)$ and $PM_2(G)$, respectively. We can see that, in the given domain, $PM_2(G)$ and $PM_1(G)$ are identical vertical lines.

First and second Zagreb polynomial of HX_n are computed as:

$$M_1(G,x) = \sum_{uv\in E(G)} x^{[d_u+d_v]}$$

$$M_1(HX_n,x) = \sum_{uv\in E_1} x^{[d_u+d_v]} + \sum_{uv\in E_2} x^{[d_u+d_v]} + \sum_{uv\in E_3} x^{[d_u+d_v]} + \sum_{uv\in E_4} x^{[d_u+d_v]} + \sum_{uv\in E_5} x^{[d_u+d_v]}$$

$$M_1(HX_n,x) = (|E_1(HX_n)|)x^7 + (|E_2(HX_n)|)x^9 + (|E_3(HX_n)|)x^8 + (|E_4(HX_n)|)x^{10} + (|E_5(HX_n)|)x^{12}$$
$$= (12)x^7 + (6)x^9 + (6n-18)x^8 + (12n-24)x^{10} + (9n^2-33n+30)x^{12}$$
$$M_2(G,x) = \sum_{uv\in E(G)} x^{[d_u\times d_v]}$$
$$M_2(HX_n,x) = \sum_{uv\in E_1} x^{[d_u\times d_v]} + \sum_{uv\in E_2} x^{[d_u\times d_v]} + \sum_{uv\in E_3} x^{[d_u\times d_v]} + \sum_{uv\in E_4} x^{[d_u\times d_v]} + \sum_{uv\in E_5} x^{[d_u\times d_v]}$$
$$M_2(HX_n,x) = (|E_1(HX_n)|)x^{12} + (|E_2(HX_n)|)x^{18} + (|E_3(HX_n)|)x^{16} + (|E_4(HX_n)|)x^{24} + (|E_5(HX_n)|)x^{36}$$
$$= (12)x^{12} + (6)x^{18} + (6n-18)x^{16} + (12n-24)x^{24} + (9n^2-33n+30)x^{36}$$

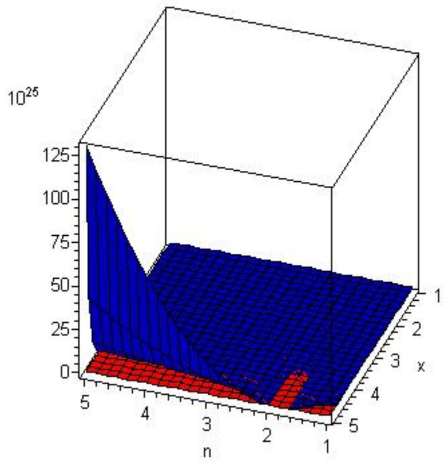

Fig. 12 – First and second Zagreb polynomial $M_1(G, x)$ and $M_2(G, x)$ of G equivalent to HX_n , Red and Blue colors represent $M_1(G, x)$ and $M_2(G, x)$, respectively. We can see that, in thegiven domain, $M_2(G, x)$ is more dominating than $M_1(G, x)$.

CONCLUSION

In this paper, we consider different networks and discuss different variants of Zagreb indices and Zagreb polynomials are analyzed for these networks using edge partition based on degree of vertices of the edges of the corresponding chemical graphs. We found exact relations of forgotten topological index, First Zagreb index, second Zagreb index, hyper Zagreb index, multiplicative Zagreb indices as well as Zagreb polynomials for different networks. In future, we are interested to found some new chemical compounds and then study their topological indices which will be quite helpful to understand their underlying pologies.

Funding: This research is supported by Anhui Provincial Natural Science Foundation Project Number: KJ2020A0780.

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