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COMPARISON BETWEEN GAUSSIAN AND ADVECTION DIFFUSION EQUATION OF EULERIAN AND LAGRANGIAN INTEGRAL TIME SCALES IN STABLE CONDITION

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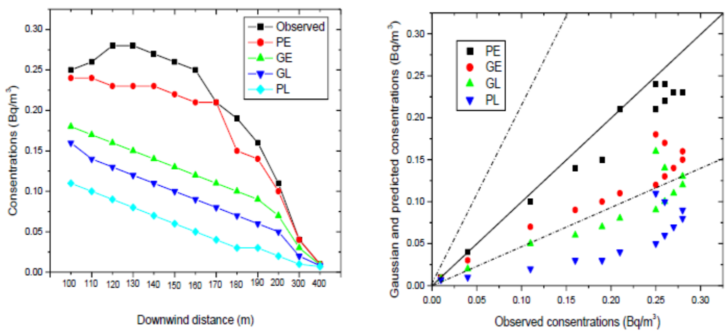

The displacement mean-square parameters $\overline{X^2}(t)$, $\overline{Y^2}(t)$ and $\overline{Z^2}(t)$ need to be increasing functions of time. In a stable and homogeneous field of turbulence, the variances of Lagrangian velocity fluctuations must be equal to the corresponding Eulerian velocity variances and independent of time. The mean-square particle displacement rises proportionately to the square of the diffusion time t for small diffusion times. The mean-square diffusion finally becomes proportional to the diffusion time (t) and the Lagrangian integral time scale (T_{iL}) for long diffusion times. The three-dimensional advection-diffusion equation and Gaussian model are computed using these dispersion values. The results of these concentrations are contrasted with Iodine-131 experimental data that was gathered at the Egyptian Atomic Energy Authority under steady conditions.

INTRODUCTION

The new approach does provide considerably more physical insight into the diffusion phenomenon and some useful information on diffusivities associated with the Brownian diffusion will not be covered here, the reader may refer to the original references given above or the more recent reviews.[1–3]

In this paper, now the advection-diffusion equation in two dimensions will be solved by second mathematical model, considering the height of Atmospheric Boundary Layer (ABL) (h) is discretized into N sub-interval layers such that within each interval, k_z and u are taken as average value. Then the solution the advection-diffusion equation in two dimensions is reduced to the solutions of N equations, Then, the concentration in three dimensions is taken.[4]

The effect of any uniform mean motion is simply to transport particles at a fixed speed in the direction of flow. Let us consider the Lagrangian motion of a

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fluid particle initially located or released at the origin in the above frame of reference. Although the Lagrangian motion is three dimensional, following Taylor,[5] we can focus on particle movements in one direction (say, in the Y direction) only. The particle displacement Y from its initial position at the origin is only a function of time t after release. This is related to the Lagrangian turbulent velocity $\acute{V}(t)$ as:

$$\acute{V}(t) = \frac{dY(t)}{dt} \qquad Y(t) = \int_0^t \acute{V}(\acute{t})d\acute{t} \tag{1}$$

The simplest meaningful statistical measures of dispersion, we can compute the mean square particle displacement $\overline{X^2}(t)$, $\overline{Y^2}(t)$ and $\overline{Z^2}(t)$ which following the above reasoning must be increasing functions of time. This is the variances of Lagrangian velocity fluctuations, which must be independent time in a stationary and homogeneous field of turbulence and equal to the Eulerian velocity variances. The rate of change of the mean-square particle displacement $\overline{Y^2}(t)$ with time can be related to the statistics of turbulence, using Eq. (1) and Reynolds's averaging rules as:

$$\frac{d\overline{Y^2}}{dt} = 2\bar{Y}(t)\frac{d\bar{Y}}{dt} = 2\left[\int_0^t \acute{V}(\acute{t})d\acute{t}\right]\acute{V}(\acute{t}) = 2\overline{\int_0^t \acute{V}(t)\,\acute{V}(\acute{t})d\acute{t}} \tag{2}$$

In a stationary field of turbulence, the average of the velocity product in Eq. (2) will depend on the time difference $\xi = \acute{t} - t$ only and may be replaced by $\overline{\acute{V}^2}R_L(\xi)$, so that:

$$\frac{d\overline{Y^2}}{dt} = 2\overline{\acute{v}^2}\int_0^t R_L(\xi)d\xi \tag{3}$$

where $R_L(\xi)$ is the autocorrelation coefficient of Lagrangian velocity.

Taylor[5] obtained the fundamental results the covariance between particle velocity and displacement as follows:

$$\overline{\acute{V}(t)Y(t)} = \frac{1}{2}\frac{d\overline{Y}^2}{dt} = \overline{\acute{v}^2}\int_0^t R_L(\xi)d\xi \tag{4}$$

Integrating of Eq. (3) yields an alternative "Law of diffusion"

$$\overline{Y^2}(t) = 2\overline{\acute{v}^2}\int_0^t\int_0^{\acute{t}} R_L(\xi)d\xi d\acute{t} \tag{5}$$

Similar expression can be written for $\overline{X^2}(t)$ and $\overline{Z^2}$. Taylor[5] called Eq. (5) "rather remarkable" because it reduces the problem of diffusion in a simplified type of turbulent motion to the consideration of a single quantity namely, the correlation coefficient between the velocity of a particle at one instant and that a time ξ later.

For small diffusion time: $t << T_{iL}, R_L(\xi)$ does not differ appreciably from unity, so that Eqs. (3) and (5) can be approximated to:

$$\frac{d\overline{Y^2}}{dt} = 2t\overline{\acute{v}^2} \tag{6}$$

$$\overline{Y^2}(t) = \overline{\acute{v}^2}t^2 \tag{7}$$

indicating that initially mean-square particle displacement increases in proportion t^2.

For large diffusion time: $t >> T_{iL}$., one would expect the integral in Eq. (3) to approach a constant value, equal to the Lagrangian integral time scale T_{iL}, so that Eqs. (3) and (5) may be approximated to:

$$\frac{d\overline{Y^2}}{dt} = 2T_{iL}\overline{\acute{v}^2} \tag{8}$$

$$\overline{Y^2}(t) = 2\overline{\acute{v}^2}T_{il}t \tag{9}$$

indicating that: the mean-square diffusion becomes proportional to t, the equations can also be written in terms of the standard deviation of particle displacement or so called one-particle diffusion parameter $\sigma_y = \left(\overline{Y^2}\right)^{0.5}$ as

$$\sigma_y = \sigma_v t \quad \text{for} \quad t<<T_{iL} \tag{10}$$

$$\sigma_y = \sqrt{2}\sigma_v(T_{iL}t)^{0.5} \quad \text{for} \quad t>>T_{iL} \tag{11}$$

Taylor[5] also noted that, when $t>>T_{iL}$, the covariance between the particle displacement and its velocity $\overline{Y\acute{V}}$, becomes constant in spite of the fact that $\overline{Y^2}$ continually increase. It obviously follows from Eqs. (4) and (8)

$$\overline{Y\acute{V}} = \frac{1}{2}\frac{d\overline{Y^2}}{dt} = \overline{\acute{v}^2}T_{iL} \quad \text{at} \quad t>>T_{iL} \tag{12}$$

which implies that particle displacement must always be positively correlated with particle velocity, but the correlation coefficient must decrease with increasing diffusion time and, hence, with increasing $\overline{Y^2}$ since the above covariance or $0.5\, d\overline{Y^2}/dt$, will be shown equivalent to eddy diffusivity K_y, so Eq. (12) can also be consider as an expression for eddy diffusivity as a product of the variance of velocity and the integral time scale.

Taylor[5] assumed the same exponential form for $R_L(\xi)$ in deriving the following general expression for the diffusion parameter.

$$\sigma_y = \sigma_v T_{iL}\left[\frac{2t}{T_{il}} - 2\left(1 - e^{-\frac{t}{T_{iL}}}\right)\right]^{1/2} \tag{13}$$

Note that the exponential form of the autocorrelation coefficient is as follows:

$$R_L(\xi) = \exp\left(-\frac{|\xi|}{T_{iL}}\right) \tag{14}$$

In alternative form of the autocorrelation function which satisfies the above constraint is

$$R_L(\xi) = \exp\left(-\frac{\pi}{4}\frac{\xi^2}{T_{iL}^2}\right) \quad (15)$$

Which implies the following expression for the dispersion parameter

$$\sigma_y = \sigma_v T_{iL}\left[\frac{2t}{T_{il}} erf\left(\frac{\sqrt{\pi}}{2}\frac{t}{T_{iL}}\right) + \frac{4}{\pi}\exp\left(\frac{-\pi}{4}\frac{t^2}{T_{iL}^2} - \frac{4}{\pi}\right)\right]^{1/2} (16)$$

A useful simplification of Taylor is theorem is obtained by performing the integration of Eq. (5) with respect to t by parts using Leibnitz's rule

$$\overline{\sigma_y{}^2}(t) = 2\overline{\acute{v}^2}\int_0^t (t-\xi)\, R_L(\xi) d\xi \quad (17)$$

This result can also be written in terms of the normalized Lagrangian spectrum function $S_L(n)$ by using Fourier transfer relations for formula:

$$R_L(\xi) = \int_0^\infty S_L(n)\cos(2\pi n\xi)\, dn$$

One gets:

$$\overline{\sigma_y{}^2}(t) = 2t^2\overline{\acute{v}^2}\int_0^\infty S_L(n)\frac{\sin^2(\pi nt)}{(\pi nt)^2} dn \quad (18)$$

Similar expressions can be obtained for $\overline{\sigma_x{}^2}(t)$ and $\overline{\sigma_z{}^2}(t)$, provided turbulence is homogeneous in those directions also, for a completely homogeneous field of turbulence. Batchelor[6] derived three dimensional versions of these results. For simplicity, we discuss here only one-dimensional relations, which describe dispersion in one direction at a time and are often used in applications to atmospheric dispersion modeling.

Following the simplifying notation of the effect of smoothing of time series, which can be expressed from Eq. (18) as:

$$\overline{\sigma_y{}^2}(t) = 2t^2\overline{V^2} \quad (19)$$

where $\overline{V^2}$ represents the variance of the smoothed Lagrangian velocity with smoothing interval of t. Eq. (19) can also be interpreted as a simple but general relationship for dispersion parameter σ_y, as a product of diffusion time t and root-mean square of particle velocity averaged over t Ogura[7] *i.e.* $\sigma_y = \overline{V'}t$.

Hay and Pasquil[8] sought to express the above dispersion relations entirely in terms of fixed point (Eulerian) statistics. The basis of their modification is the simple hypothesis that the forms of $R_L(\xi)$ and $R_E(\tau)$ are similar as follows:

$$R_L(\xi) = R_E(\tau) \text{ for } \xi = \beta\tau \quad (20)$$

where $\beta = T_{iL}/T_{iE}$ is the ratio of Lagrangian and Eulerian integral scales. The assumption of similarity between Lagrangian and Eulerian autocorrelation coefficients, also, implies the similarity between their corresponding spectral functions as:

$$nS_L(n) = \beta n S_E(\beta n) \quad (21)$$

which follows from the appropriate Fourier transform relations

$$S(n) = 4\int_0^\infty R(\tau)\cos(2\pi n\tau)\, d\tau$$

$$R(\tau) = \int_0^\infty S(n)\cos(2\pi n\tau)\, dn$$

between autocorrelation and spectrum functions.

Substituting from Eq. (21) into Eq. (18), one obtains:

$$\overline{\sigma_y{}^2}(t) = \overline{\acute{v}^2}t^2\int_0^\infty S_E(n)\frac{\sin^2(\pi n\acute{t})}{(\pi n\acute{t})^2} dn \quad (22)$$

where $\acute{t} = t/\beta$ may be considered as a modified diffusion or travel time. Note that Eq. (22) is similar to Eq. (18) but involves the more easily measured spectrum of Eulerian velocity. For the reduced variance of a smoothed time series Eq. (22) can also be written as:

$$\overline{\sigma_y{}^2}(t) = 4\overline{v_t'^2}t^2 T_E \quad (23)$$

where $\acute{t} = t/\beta$ is now the appropriate averaging time or smoothing interval, $S_E(n) = 4T_E$. Although Eq. (23) is quite similar to Eq. (19), it involves the more easily obtainable variance of Eulerian velocity after appropriate smoothing.

EFFECTS OF FINITE SAMPLING AND RELEASE TIMES

The effect of finite sampling duration on spectrum and variance of stationary variable (time series) has been discussed before. One can easily write the finite-sampling analogs of the statistical theory relations, for example, from Eq. (5) and Eq. (18) respectively

$$\overline{\sigma_{y_T}^2} = 2\overline{\acute{v}_T^2}\int_0^t\int_0^{\acute{t}} R_L(\xi) d\xi d\acute{t} \quad (24)$$

$$\overline{\sigma_{y_T}^2} = \overline{\acute{v}_T^2}t^2\int_0^t S_L(n)\frac{\sin^2(\pi nt)}{(\pi nt)^2} dn = \overline{\acute{V}_{T,t}^2}t^2, \quad (25)$$

where the first subscript refers to the finite sampling duration and the second subscript indicates the finite smoothing or averaging interval.

The Hay-Pasquill approach described earlier yields the following dispersion relation entirely in terms of the fixed-point Eulerian velocity:

$$\overline{\sigma_{y_T}^2} = \overline{\acute{v}^2}t^2\int_0^\infty S_E(n)\frac{\sin^2(\pi n\acute{t})}{(\pi n\acute{t})^2}\left[1 - \frac{\sin^2(\pi nT)}{(\pi nT)^2}\right] dn \quad (26)$$

This can be written as:

$$\overline{\sigma_{y_T}^2} = \overline{\acute{v}_{T,\acute{t}}^2}t^2 \quad (27)$$

where $\acute{t} = t/\beta$ represents the appropriate interval for smoothing or averaging the finite-duration time series $\acute{v}(t)$ before determining its variance.

RELATION BETWEEN CONCENTRATION FIELD AND DISPERSION PARAMETERS

For a continuous point source, the average concentration at a point (*x*, *y*, *z*) is equivalent to the probability of finding a particle at (*x*, *y*, *z*) at any time. Considering contribution from all particles travel times, one can write

$$\bar{c}(x,y,z) = Q\int_0^{\infty} \rho(x,y,z;t)dt \qquad (28)$$

where Q is the emission rate of point source. With the Gaussian form of probability density functions of particle displacement in the *y* and *z* directions, one would expect from Eq. (28) the concentration distributions in *y* and *z* directions to be Gaussian also. It is easy to see that the width of two distributions must be identical dispersion parameters σ_y and σ_z also represent the standard deviations of concentration distributions across the plume in the *y* and *z* directions respectively, that is

$$\sigma_y = \left[\int_{-\infty}^{\infty} \bar{c}(x,y,z)y^2\,dy/\bar{c}(x,y,z)dy\right]^{1/2}$$

$$\sigma_z = \left[\int_{-\infty}^{\infty} \bar{c}(x,y,z)\,z^2 dz/\bar{c}(x,y,z)dz\right]^{1/2} \qquad (29)$$

The process of diffusion by continuous movements implies an apparent turbulent diffusivity. This work has been examined in detail by Batchelor[6] and Csanady[2]. Starting from Gaussian expression for concentration distribution in a slender plume, resulting from a continuous point source in an infinite, homogeneous medium that is:

$$\bar{c}(x,y,z) = \frac{Q}{2\pi\bar{U}\sigma_y\,\sigma_z}\exp\left[\frac{-y^2}{2\sigma_y^2} - \frac{(z+H)^2}{2\sigma_z^2} - \frac{(z-H)^2}{2\sigma_z^2}\right] \quad (30)$$

where *H* is the effective height, $H = h_s + 3\frac{w}{U}D$; *w* is the exist velocity, D is the diameter of the stack and U is the wind speed at stack height. One can show that the advective transport of material by mean flow is given by

$$\bar{u}\frac{\partial c}{\partial x} = \frac{\bar{u}}{2}\frac{d\sigma_y^2}{dx}\frac{\partial^2 c}{\partial y^2} + \frac{\bar{u}}{2}\frac{d\sigma_z^2}{dx}\frac{\partial^2 c}{\partial \sigma_z^2} \qquad (31)$$

Equation (31) can be recognized as being similar to the approximate diffusion equation

$$\bar{u}\frac{\partial c}{\partial x} = K_y\frac{\partial^2 c}{\partial y^2} + K_z\frac{\partial^2 c}{\partial \sigma_z^2}\,, \qquad (32)$$

in which diffusion in the direction of mean flow has been ignored. In fact, the two equations would be identical if eddy diffusivities were expressed in terms of dispersion parameters as:

$$K_y = \frac{u}{2}\frac{d\sigma_y^2}{dx} = \frac{1}{2}\frac{d\sigma_y^2}{dt}$$

$$K_z = \frac{u}{2}\frac{d\sigma_z^2}{dx} = \frac{1}{2}\frac{d\sigma_z^2}{dt} \qquad (33)$$

in which we have used the usual transformation between travel time and distance from the source. From Eq. (3), these apparent eddy diffusivities can also be expressed as:

$$K_y = \acute{v}^2\int_0^t R_{Lv}(\xi)\,d\xi$$

$$K_z = \acute{w}^2\int_0^t R_{Lw}(\xi)\,d\xi \qquad (34)$$

Similar expression for eddy diffusivities as given in Eq. (33), with the addition of $K_x = \frac{1}{2}\frac{d\sigma_x^2}{dt}$, can be derived for puff diffusion Casnady[2], if one requires that the Gaussian puff formula for an instantaneous point source in an infinite medium satisfy the mean diffusion equation.

Taylor's theorem gives the following useful relations between eddy diffusivities and other turbulence statistics:

$$K_x = \acute{u}^2 T_{Lu};\;\; K_y = \acute{v}^2 T_{Lv};\; K_z = \acute{w}^2 T_{Lw} \quad (35)$$

which are applicable only in the limit of large diffusion time $t >> T_{iL}$.

The scaling factor for the *i*-velocity component (*u*, *v*, *w*) defined as the ratio between Lagrangian and Eulerian time scales from Degrazia[9] as follows:

$$B_i = \frac{T_{Li}}{T_{Ei}} = \frac{0.55U}{\sigma_i} \qquad (36)$$

The Lagrangian time scales T_{L_i} can be written in the form in stable condition Mangia[10] as follows:

$$T_{L_v} = \frac{0.25z}{\left(1-\frac{z}{h}\right)\left(1+\frac{3.7z}{L(1-z/h)^{1.25}}\right)u_*} \qquad (37)$$

$$T_{L_w} = \frac{0.15z}{\left(1-\frac{z}{h}\right)\left(1+\frac{3.7z}{L(1-z/h)^{1.25}}\right)u_*} \qquad (38)$$

where: u_* is the friction velocity, L is the Monion-Obukhov length scale and h is the convective boundary layer (CBL). The Lagrangian variance of particle position is given by Eq. (9):

$$\bar{X}_i^2 = 2u_i^2 t T_{iL} \qquad (39)$$

where: $t = x/U$ is the diffusion travel time, U is the horizontal mean wind speed and $u_i^2 = \sigma_i^2$ are the velocity variances.

In the stable condition, the velocity variances can be obtained from the u_* Stull[11], so that $\sigma_w = 1.3u_*$ and $\sigma_v = 1.9u_*$.

Then, the vertical dispersion parameter in Lagrangian case can be written from Eq. (39) as follows:

$$\sigma_z^2 = 0.507\frac{u_* xz}{U}\frac{L\left(1-\frac{z}{h}\right)^{0.25}}{L\left(1-\frac{z}{h}\right)^{1.25}+3.7z} \qquad (40)$$

Also, the lateral dispersion parameter in Lagrangian case can be written from Eq. (39) as follows:

$$\sigma_y^2 = 1.805\frac{u_* xz}{U}\frac{L\left(1-\frac{z}{h}\right)^{0.25}}{L\left(1-\frac{z}{h}\right)^{1.25}+3.7z} \quad (41)$$

Equations (40) and (41) are used in Eq. (30) at y = 0 to get the Gaussian plume model at centerline $\bar{c}(x,0,z)$as follows:

$$\bar{c}(x,0,z) = \frac{Q}{2\pi\bar{U}\sigma_y\,\sigma_z}\exp\left[-\frac{(z+H)^2}{2\sigma_z^2} - \frac{(z-H)^2}{2\sigma_z^2}\right]\exp\left(-\frac{\nu x}{U}\right) \quad (42)$$

where: $e^{-\frac{\nu x}{u}}$ is the radioactive decay for the specified nuclide, ν =9.95 × $10^{-7}\mathrm{s}^{-1}$.

Also, from Eq. (36), the Eulerian time scales T_{E_i} can be written in the form

$$T_{E_v} = T_{L_v}\sigma_v/0.55U \quad (43)$$

Substituting from Eq. (37) in Eq. (43), one gets:

$$T_{E_v} = \frac{0.864}{U}\frac{zL\left(1-\frac{z}{h}\right)^{0.25}}{L\left(1-\frac{z}{h}\right)^{1.25}+3.7z} \quad (44)$$

Also, substituting from Eq. (38) in Eq. (43), to get the vertical Eulerian time scales T_{E_W}as follows:

$$T_{E_W} = \frac{0.355}{U}\frac{zL\left(1-\frac{z}{h}\right)^{0.25}}{\left[L\left(1-\frac{z}{h}\right)^{1.25}+3.7z\right]} \quad (45)$$

The Eulerian variance of particle position is given by Eq. (23) as follows:

$$\bar{X}_i^2 = 4u_i^2 t^2 T_{E_i} \quad (46)$$

Then, the vertical dispersion parameter in Eulerian case can be written from Eq. (46) as follows:

$$\sigma_z^2 = 2.40\frac{x^2u_*^2}{U^3}\frac{zL\left(1-\frac{z}{h}\right)^{0.25}}{\left[L\left(1-\frac{z}{h}\right)^{1.25}+3.7z\right]} \quad (47)$$

Also, the lateral dispersion parameter in Eulerian case can be written from Eq. (46) as follows:

$$\sigma_y^2 = \frac{12.48x^2u_*^2}{U^3}\frac{zL\left(1-\frac{z}{h}\right)^{0.25}}{\left[L\left(1-\frac{z}{h}\right)^{1.25}+3.7z\right]} \quad (48)$$

Substituting from Eqs. (47) and (48) are used in Eq. (42) at y = 0 to get the Gaussian plume model at centerline $\bar{c}(x,0,z)$.

Mathematical model. The diffusion equation in three dimensions is

$$u\frac{\partial C(x,y,z)}{\partial x} = \frac{\partial}{\partial y}\left(k_y\frac{\partial C(x,y,z)}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z\frac{\partial C(x,y,z)}{\partial z}\right) \quad (49)$$

where: $C(x,y,z)$ is the concentration of pollutants (g/m^3) or (Bq/m^3), k_y and k_z are the eddy diffusivities in crosswind and vertical direction respectively, $u$is the wind speed (m/s), x is downwind distance (m).

By taking crosswind integration with respect to y from –∞ to ∞, one gets diffusion equation in two dimensions as follows:

$$u\frac{\partial C_y(x,z)}{\partial x} = \frac{\partial}{\partial z}\left(k_z\frac{\partial C_y(x,z)}{\partial z}\right) \quad (50)$$

where: $C_y(x,z)$ is the crosswind integrated concentration of pollutants. Equation (50) is solved under the boundary conditions as follows:

(a) The condition of null flux is applied on the ground surface and at the mixing height

$$k_z\frac{\partial C}{\partial z} = 0 \quad \text{at} \quad z = 0, h. \quad (51a)$$

(b) The mass continuity is used

$$uC_y(0,z) = Q\delta(z-h) \quad \text{at} \quad x = 0 \quad (51b)$$

where: h is the height of the atmospheric boundary layer (ABL)(m), Q is the emission rate(g/s) or (Bq), and δ is a Dirac delta function.

(c) The crosswind integrated concentration tends to zero as z tends to ∞

$$C_y(x,z) \to 0 \qquad \text{as } z \to \infty \quad (51c)$$

(d)The crosswind integrated concentration vanishes at the mixing height

$$C_y(x,z) = 0 \qquad \text{at } z = h \quad (51d)$$

Now the advection-diffusion equation in two dimensions Eq. (50) will be solved by second mathematical model, considering the height of ABL (h) is discretized into N sub-interval layers such that within each interval, $k_z$and u are taken as average values. Then the solution of Eq. (50) is reduced to the solutions of N equations of the following type

$$u_i\frac{\partial C_y(x,z)}{\partial x} = k_i\frac{\partial^2 C_y(x,z)}{\partial z^2} \quad (52)$$

where:

$$k_i = \frac{1}{z_{i+1} - z_i}\int_{z_n}^{z_{i+1}} k_i(z)\,dz$$

$$u_i = \frac{1}{z_{i+1} - z_i}\int_{z_i}^{z_{i+1}} u_i(z)\,dz$$

for $z_i \le z \le z_{i+1}$, i = 1 : N

Then, the concentration in three dimensions is taken from Essa[4]

$$C(x,y,z,h) = \frac{Q}{2\sqrt{2x}\,\pi\,\sigma_y u_i}e^{-\frac{(z-h)^2u_i}{4k_i x} - \frac{y^2}{2\sigma_y^2} - \frac{\nu x}{u}} \quad (53)$$

where: k_i are taken from two equations (40) and (47) in Lagrangian and Eulerian respectively; $\sigma_y$is the standard deviation in y direction and $e^{-\frac{\nu x}{u}}$ is the radioactive decay for the specified nuclide, ν is radioactive coefficient.

RESULTS AND DISCUSSION

The studies conducted in stable conditions at the Egyptian Atomic Energy Authority's Second Research Reactor in Inshas yielded the observed amounts of I^{131} isotope. The samples that were seen came from a 27 meter-tall stack with a 0.6 meter roughness length. The weather information collected during the tests is taken into account Essa,[12] and it is included in Table 1. Table 2 also includes the expected and observed amounts of I^{131} isotope below the plume centerline from Eqs. 42 and 53. Figures 1 and 2 demonstrate that the anticipated concentrations and observed concentrations accord well, with the predicted values being within a factor of two of the observed data.

Table 1

Shows the meteorological conditions during the experiments in stable Essa[12]

Exp	Atmospheric stability	*L*	u_*(m/s)	u_{27}(m/s)	mixing height *h* (m)
5	E	55	0.50	3.80	209

Table 2

Measured and proposed concentrations of I^{131} in stable condition

Run	Distance (m)	Observed *C* (Bq/m^3)	Gaussian Lagrangian *C*(Bq/m^3)	Predicted Lagrangian *C*(Bq/m^3)	Gaussian Eulerian *C*(Bq/m^3)	Predicted Eulerian *C*(Bq/m^3)
1	100	0.25	0.16	0.11	0.18	0.24
2	110	0.26	0.14	0.1	0.17	0.24
3	120	0.28	0.13	0.09	0.16	0.23
4	130	0.28	0.12	0.08	0.15	0.23
5	140	0.27	0.11	0.07	0.14	0.23
6	150	0.26	0.1	0.06	0.13	0.22
7	160	0.25	0.09	0.05	0.12	0.21
8	170 180	0.21	0.08	0.04	0.11	0.21
9	180	0.19	0.07	0.03	0.1	0.15
10	190	0.16	0.06	0.03	0.09	0.14
11	200	0.11	0.05	0.02	0.07	0.1
12	300	0.04	0.02	0.01	0.03	0.04
13	400	0.01	0.01	0.01	0.01	0.01

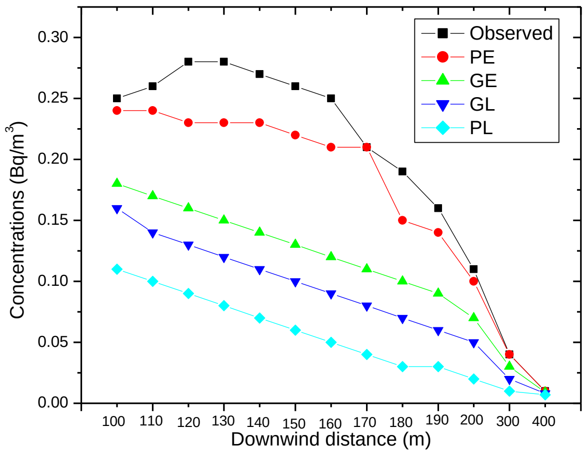

Fig. 1 – The variations of observed, Gaussian and proposed concentrations models *via* downwind distance (m) in Lagrangian and Eulerian forms.

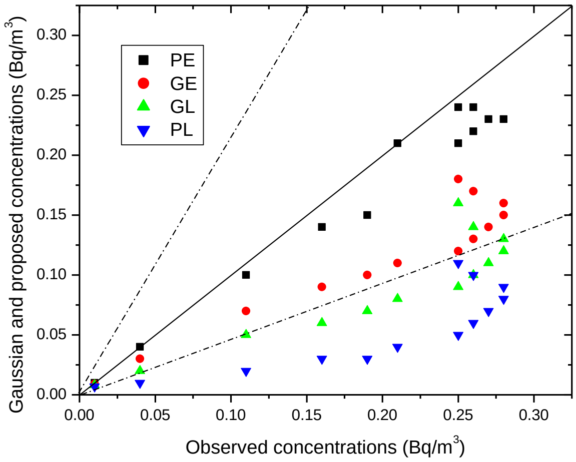

Fig. 2 – Shows the variations of Gaussian and proposed models *via* observed concentrations in Lagrangian and Eulerian forms.

Figure 1 illustrates how the suggested and Gaussian concentrations in Eulerian form are closer to the observed I^{131} concentration data than they are in Lagrangian form. All of the Gaussian and the proposed using Eulerian form fell within a factor of two, but as seen in Fig. 2, the Gaussian in Lagrangian form fell within a factor of three, and the proposed in Lagrangian form fell within a factor of four.

Statistical Technique

Comparing between Gaussian, proposed and observed concentrations is introduced Hanna,[13] where; NMSE is the Normalized Mean Square Error, FB is the Fraction Bias, COR is the Correlation coefficient and FAC2 is the Factor of Two.

Table 3

Comparison between Gaussian, proposed and observed concentrations using Lagrangian and Eulerianin stable condition

	NMSE	FB	COR	FAC2
Gauss. Lagrangian	0.86	0.77	0.91	0.44
Proposed Lagrangian	2.3	1.1	0.83	0.27
Proposed Eulerian	0.03	0.13	0.99	0.88
Gaussian Eulerian	0.42	0.55	0.95	0.57
Previous work Essa and El Said (2023) h=27m	0.008	0.05	0.96	0.97

Table 3 shows that the entire proposed model and Gaussian model in Eulerian space fell within a factor of two when paired with the measured concentration data for I^{131}. Gaussian and proposed Lagrangian models, on the other hand, were not within a factor of two. Moreover, statistical data suggests that the models that are suggested for Eulerian frameworks are better than those for Lagrangian frameworks. Moreover, the proposed of the Eulerian model is better than the Gaussian of the Eulerian model. Additionally, Essa and El Said's[14] earlier reference demonstrates the suitability of the statistics at h = 27 m.

CONCLUSIONS

With respect to the measured concentration data of I^{131}, the whole suggested model and Gaussian model in Eulerian fell within a factor of two. However, Gaussian and suggested Lagrangian models were outside of a factor of two. Furthermore, statistical evidence indicates that the proposed Gaussian models in Eulerian frameworks are superior to those in Lagrangian frameworks. Furthermore, the Eulerian model's Gaussian is superior to the Lagrangian model's Gaussian.

Compared to the proposed and Gaussian concentrations in Lagrangian form, the proposed concentrations in Eulerian and its Gaussian form are closer to the observed concentration data of I^{131}. All of the Gaussian and the suggested using the Eulerian form fell within a factor of two, whereas the proposed in the Lagrangian form and the Gaussian in the Lagrangian form fell within a factor of three and four, respectively.

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