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URBAN DIFFUSION AND AIR QUALITY MODELS

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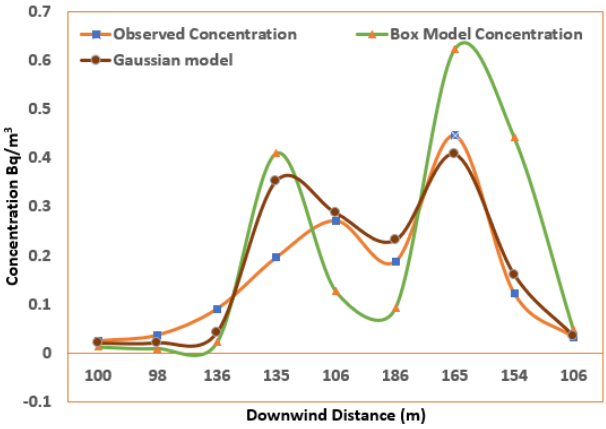
In this research, the Atmospheric Turbulence and Diffusion Laboratory (ATDL) is another sample of Gaussian-based urban diffusion models. Also, the simplest box model without chemical transformation can be driven to get average concentration using the lateral and vertical standard deviations values of Brookhaven National Laboratory through different stabilities. The two models (Gaussian and Box models) are applied to obtain the concentration in three dimensions to compare with experimental data of Iodine-135 (I^{135}). Also, using the two models to obtain the normalized crosswind concentrations to compare with data from Copenhagen-Denmark for Sulphur-Hexi-Fluoride (SF_6).



INTRODUCTION

A different diffusion models are used to estimate the pollutants concentration from urban sources. Most urban air quality models also predict concentrations of secondary pollutants resulting from photochemical reactions in urban air during transport and dispersion. The range from easy practice models to imaginary three-dimensional urban models. Here we review only Gaussian and box models of urban diffusion air quality.

The atmospheric diffusion models that are widely used for regulatory purposes have been reviewed by World Meteorological Organization (WMO). The most important parameters predicted by these models are the magnitude and location of the maximum ground level concentration. The Gaussian Plume Model (GPM) is a widely used dispersion model in more regulatory applications than the other models.

Simpler urban diffusion models apply the same Gaussian plume formulation to multiple sources of pollution in an urban area. Respect to the distributed nature of these sources, however a numerical model becomes necessary when an analytical formulation is used for each individual point source or each grid element of the area source emissions.

Turbulent from the biggest point sources can be obtained contamination such Co and SO_2 which are emitted from point sources. If the flow rate is not inside a factor of two, then the turbulent model has error before it begins working.

In this research, the two models (Gaussian and Box models) are applied to obtain the concentration

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in three dimensions to compare with experimental data of Iodine-135 (I^{135}). Also, using the two models to obtain the normalized crosswind concentrations to compare with data from Copenhagen-Denmark for Sulphur Hexi-Fluoride (SF_6).

Gaussian Plume models

ATDL to be example of Gaussian-based on urban diffusion models was used.[1,2] The ATDL models' formulation for area source has been incorporated in the United State, Environmental protection Agency U.S. EPA's (1986–1996) used air quality model RAM. The mean concentration at any receptor points where the origin of the coordinates is obtained by integration.

$$\bar{C}_0 = \int_0^{\infty}\int_{-\infty}^{\infty} \frac{Q_a}{\pi\bar{u}\sigma_y\sigma_z}\exp\left(\frac{-y^2}{2\sigma_y^2}\right)\mathrm{d}y\mathrm{d}z \quad (1)$$

Note that the sources are located at the surface level. For further simplification, using a narrow plume hypothesis, Q_a is the area source strength which depends on the downwind distance x. Eq. (1) is simplified to

$$\bar{C}_y = \frac{(2/\pi)^{1/2}}{\bar{u}}\int_0^{\infty}\frac{Q_a}{\sigma_z}\mathrm{d}x \quad (2)$$

where, $\bar{C}_y$ is the crosswind concentration, the effective transport velocity $\bar{u}$ is assumed to be spatially uniform. Eq. (2) is obtained in a suitable form to square grid which is in urban emission. If the point is obtained in the middle of the grid square *0* with its area source strength Q_{a0} and upwind grid squares 1,2,3, ……, n spaced at intervals of Δx, have area source strengths $Q_{a1}, Q_{a2}, \ldots\ldots\ldots, Q_{an}$, the piecewise integration of Eq. (2) yields:

$$\bar{C}_y = \frac{(2/\pi)^{1/2}}{\bar{u}}\left[\int_0^{\Delta x/2}\left(\frac{Q_{a0}}{\sigma_z}\right)\mathrm{d}x + \int_{\Delta x/2}^{3\Delta x/2}\left(\frac{Q_{a1}}{\sigma_z}\right)\mathrm{d}x + \cdots\right] \quad (3)$$

To perform the above integration analytically, the vertical dispersion parameter is assumed to have the power form:

$$\sigma_z = ax^b \quad (4)$$

The parameters a and b are estimated from the urban dispersion curves for σ_z versus x for each stability class.[3] Substituting from Eq. (4) into Eq. (3) and carrying out the integration, one obtains simple analytical expression for the ground level concentration (G. L. C.).

$$\bar{C}_0 = \frac{(2/\pi)^{1/2}(\Delta x/2)}{ua(1-b)}\{Q_{a0} + \sum_{=1}^{n} Q_{ai}[(2i+1)^{1-b} - (2i-1)^{1-b}]\} \quad (5)$$

where, n is the values of grids squares which important to arrive to the upwind edge of the urban area. Eq. (5) is the best average time of order one hour for dispersion parameterization is applicable extension to longer averaging times is also early made by following a simple procedure outlined.[3]

It is good to use version of the ATDL model by using approximating the values Q_{ai} in terms by Q_{a0} in Eq. (5) as follows:

$$\bar{C}_y = A\frac{Q_{a0}}{\bar{u}} \quad (6)$$

where

$$A = \left(\frac{2}{\pi}\right)^{1/2}\frac{[\Delta x(2n+1)/2]^{1-b}}{a(1-b)} \quad (7)$$

Note that the dimensionless coefficient A depends on the upwind distance from the receptor to the edge of city $\Delta x(2n+1)/2$, and stability. It increases with increasing distance as well as with increasing stability. For distances between 5 and 20 km, the estimated values of A range between 50 and 800.[3] This simple model is compared with measured concentrations of particulate matter in several cities. However, a comparison of the observed and model-computed concentration of SO_2 suggests considerable overestimation (by a factor of 4) by the model.

To obtain the concentration as follows:

$$\bar{C} = \frac{1}{\sigma_y\sqrt{2\pi}}e^{\frac{-y^2}{2\sigma_y^2}}\bar{C}_{y1} \quad (8)$$

where,

$$\bar{C}_{y1} = B\frac{Q_{a0}}{\bar{u}} \quad (9)$$

where $B = \left(\frac{2}{\pi}\right)^{1/2}\frac{[\Delta x(2n+1)/2]^{1-b-d}}{ac(1-b-d)}$

$\bar{C}$ is the Gaussian concentration at the ground plume centerline (y = 0).

Table 1

Shows the lateral and vertical standard deviations values of Brookhaven National Laboratory through different stabilities[4]

Stability classes	Values of σ_y	Values of σ_z
A	$\sigma_y = 0.40x^{0.91}$	$\sigma_z = 0.41x^{0.91}$
B	$\sigma_y = 0.40x^{0.91}$	$\sigma_z = 0.41x^{0.91}$
C	$\sigma_y = 0.36x^{0.86}$	$\sigma_z = 0.33x^{0.86}$
D	$\sigma_y = 0.32x^{0.78}$	$\sigma_z = 0.22x^{0.78}$

where, A, B and C are extremely, modularity and slightly unstable respectively and D is neutral condition.

Box Model

A simple differential equation for the average concentration $\bar{C}$ from,[3] also the box model is shown in Fig. 1 as follows:

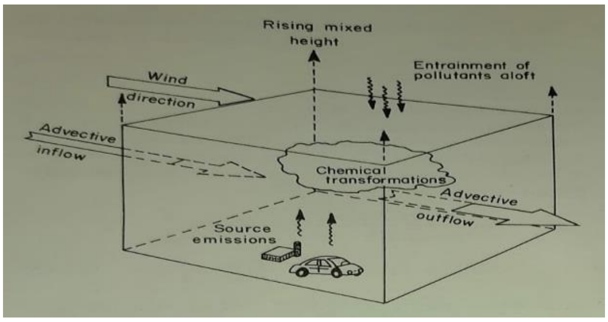

Fig. 1 – Schematic diagram showing basic elements of the box.[5]

$$\Delta x h \frac{\partial \bar{C}}{\partial t} = \Delta x Q_a + \bar{u} h(\bar{C}_b - \bar{C}) + \Delta x \frac{\partial h}{\partial t}(\bar{C}_a - \bar{C}) \quad (10)$$

where, h is convective boundary layer height, $\bar{C}_a$ is the mean concentration above a city and $\bar{C}_b$ is the mean background concentration upwind of the town. Eq. (10) becomes:

$$\frac{\partial \bar{C}}{\partial t} + \left(\frac{\bar{u}}{\Delta x} + \frac{1}{h}\frac{\partial h}{\partial t}\right)\bar{C} = \frac{Q_a}{h} + \frac{\bar{u}\bar{C}_b}{\Delta x} + \frac{1}{h}\frac{\partial h}{\partial t}\bar{C}_a \quad (11)$$

Which is obtained after substitution values of Q_a, $\bar{C}_a$, $\bar{C}_b$,u, h and $\partial h/\partial t$. Put $(\bar{C}_a = 0, \bar{C}_b = 0), \partial \bar{C}/\partial t = 0, \partial h/\partial t = 0,$ then the equilibrium concentration is obtained as follows:

$$C_e = \frac{\Delta x}{h}\frac{Q_a}{u} \quad (12)$$

This expression is similar to Eq. (6) resulting from the simplified ATDL model.

If one defines a time scale $\frac{\Delta x}{\bar{u}}$ as the flushing time required for the air to pass completely over the urban area and dimensionless time $t_* = t\bar{u}/L$, then, Eq. (10) without the background concentration terms becomes:

$$\frac{d\bar{C}}{dt^*} = \bar{C}_e - \bar{C} \quad (13)$$

Eq. (13) is solved as:

$$\bar{C} = \bar{C}_e + (\bar{C}_0 - \bar{C}_e)e^{-t_*} \quad (14)$$

where, $\bar{C}_0$ is the primary value of concentration at $t_* = 0$, which is larger than the equilibrium concentration.[6] Eq. (14) explains that the concentration decreases exponentially with increasing time. The predicting concentrations of Ozone model is established in open area of Hauston.[7]

RESULTS AND DISCUSSION

First technique concentration

The estimated data of I^{135} was calculated from experiments in convective layer were obtained from the Inshas site at the Egyptian Atomic Energy Authority (EAEA) at height 0.7 m at stack height 43 m, for 24 hours working where, the samples were gathered every 30 minutes with roughness length of 0.6 cm. The data are obtained from[8] as in Table 2. The measured concentration and proposed concentrations by Eqns. (8,9) and (12) below the plume centerlines of I^{135} isotope are presented in Table 3. Fig. 1 explains the variation of proposed and measured concentration data of I^{135} via horizontal distance in convective layer. Also, Fig. 2 shows that the variation of the proposed models with the measured concentration data.

Table 2

Shows the meteorology of nine experiments from March to May 2006 at Inshas site

Run no.	Working hours of the source	Release rate (Bq)	Wind speed (m s^{-1})	Wind direction (deg)	W_* (ms^{-1})	P-G stability class	h (m)	Vertical distance (m)
1	48	1028571	4	301.1	2.27	A	600.85	5
2	49	1050000	4	278.7	3.05	A	801.13	10
3	1.5	42857.14	6	190.2	1.61	B	973	5
4	22	471428.6	4	197.9	1.23	C	888	5
5	23	492857.1	4	181.5	0.958	A	921	2
6	24	514285.7	4	347.3	1.3	D	443	8.0
7	28	1007143	4	330.8	1.51	C	1271	7.5
8	48.7	1043571	4	187.6	1.64	C	1842	7.5
9	48.25	1033929	4	141.7	2.1	A	1642	5.0

Table 3

Shows measured, Gaussian and Box concentrations through nine experiments

Run no.	Downwind distance (m)	Observed (Bq/m^3)	Gaussian Eqns. (8,9) (Bq/m^3)	Predicted Eq.(12) (Bq/m^3)
1	100	0.025	0.011	0.043
2	98	0.037	0.020	0.032
3	136	0.091	0.042	0.099
4	135	0.197	0.622	0.179
5	106	0.272	0.287	0.141
6	186	0.188	0.643	0.539
7	165	0.447	0.407	0.326
8	154	0.123	0.446	0.218
9	106	0.032	0.060	0.017

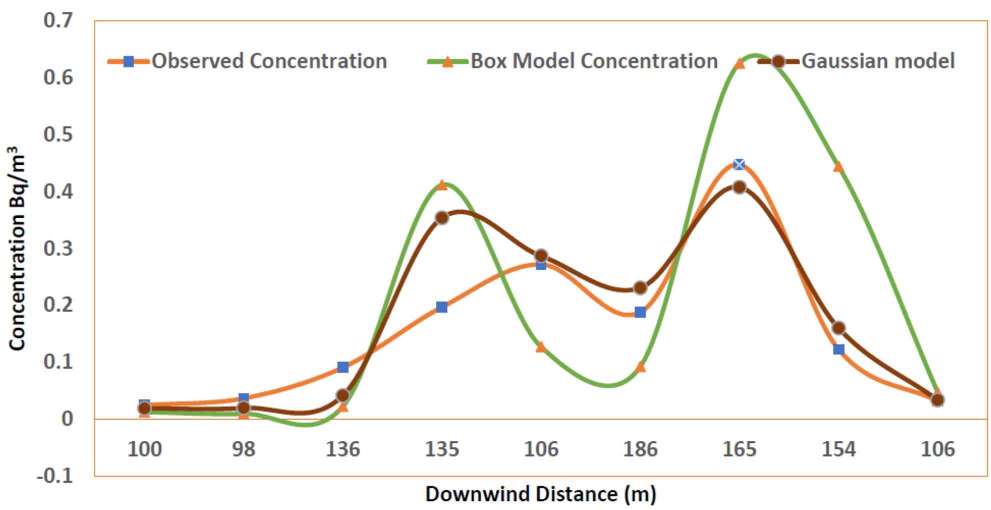

Fig. 2 – Shows the variation of Gaussian, Box model and Observation concentrations (Bq/m^3).

From Figs. 2 and 3 one, finds that The Gaussian is nearer to observed concentration of Iodine-135 (I^{135}) than the data of box model

Statistical Technique

Comparing between Gaussian, proposed and observed concentrations is introduced by.[9]

Table 4

Comparison between Gaussian and Box concentration modelsand measured concentrations in unstable condition

	NMSE	FB	COR	FAC2
Gauss. Eqs. (8,9)	0.13	–0.10	0.92	1.01
Box model Eq. (12)	0.77	–0.24	0.74	1.18

where, NMSE, FB, COR and FAC2 are the normalized mean square error, fraction bias, correlation coefficient and factor of two respectively.

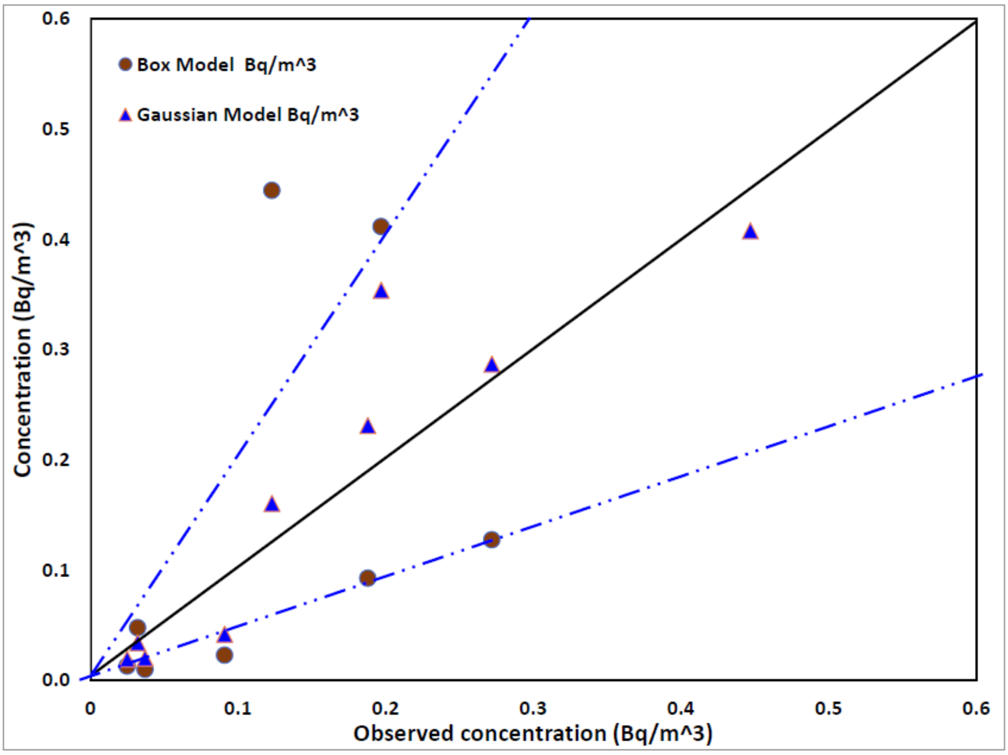

Fig. 3 – The achieving between Gaussians, Box model concentrations with observed concentration.

One finds that Table 4, that predicted models are located a factor of two with measured concentration data. Also, the Gaussian model from Eqns. (8) and (9) are better model than the Box model from Eq. (12) for NMSE, FB, COR and FAC2.

Copenhagen diffusion experiment

Second technique

In this technique we used the Copenhagen experimental technique for hex Sulphur fluoride (SF_6) which is held in the Northern part of Copenhagen, Denmark in unstable and neutral conditions. The data was released from stack height 115 m and the roughness length 0.6 m.[10]

The performance of the Gaussian and Box models are compared with measured concentration data as in Table 5. The data of Gaussian, Box models and measured concentration data via downwind distance as shown in Fig. 4. Also, the data of proposed model via measured concentration data are shown in Fig. 5.

Figure 4 represents the values of normalized crosswind Gaussian concentration are nearer to measured lateral integrated normalized concentration via distance x than normalized crosswind concentration of Box model. Figure 5 explains the variation of normalized lateral integrated of measured via Gaussian and Box model concentration. Also, from the figure one, finds that the Gaussian model located a factor of two than Box, Irwin and Briggs models.

Table 5

Observed and Normalized crosswind concentration of Gaussian and Box models by Eqns. (6,7) and (12) for C_y/Q_a, at surface of Copenhagen experiment

Run No.	P-G Stability	h (m)	w_* (ms^{-1})	U (ms^{-1})	Distance (m)	$Cy(x,0)/Q$ (10^{-4} sm^{-2}) Model assessment		
						Obs.	Gauss. Eqns. (6,7)	Box model Eq. (12)
1	A	1980	0.83	2.1	1900	6.48	8.54	4.57
1	A	1980	0.83	2.1	3700	2.31	8.37	2.22
2	C	1920	1.07	4.9	2100	5.38	4.18	2.23
2	C	1920	1.07	4.9	4200	2.95	3.86	4.46
3	B	1120	0.68	2.4	1900	8.2	7.95	7.07
3	B	1120	0.68	2.4	3700	6.22	7.94	4.13
3	B	1120	0.68	2.4	5400	4.3	7.15	6.03
4	C	390	0.71	3.1	2100	6.72	6.21	8.26
5	C	820	0.71	3.1	4200	5.84	6.61	8.26
5	C	820	0.71	3.1	6100	4.97	6.84	6.00
5	C	820	1.33	7.2	2000	3.96	2.66	2.14
6	C	1300	1.33	7.2	4200	2.22	2.85	4.49
6	C	1300	1.33	7.2	5900	1.83	2.93	6.30
6	C	1300	0.87	4.1	2000	6.7	4.67	5.27
7	B	1850	0.87	4.1	4100	3.25	4.99	5.41
7	B	1850	0.87	4.1	5300	2.23	5.10	6.99
7	B	1850	0.72	4.2	1900	4.16	4.54	5.58
8	D	810	0.72	4.2	3600	2.02	4.81	3.17
8	D	810	0.72	4.2	5300	1.52	4.98	3.12
8	D	810	0.98	5.1	2100	4.58	3.77	1.97
9	C	2090	0.98	5.1	4200	3.11	4.02	3.94
9	C	2090	0.98	5.1	6000	2.59	4.15	5.63

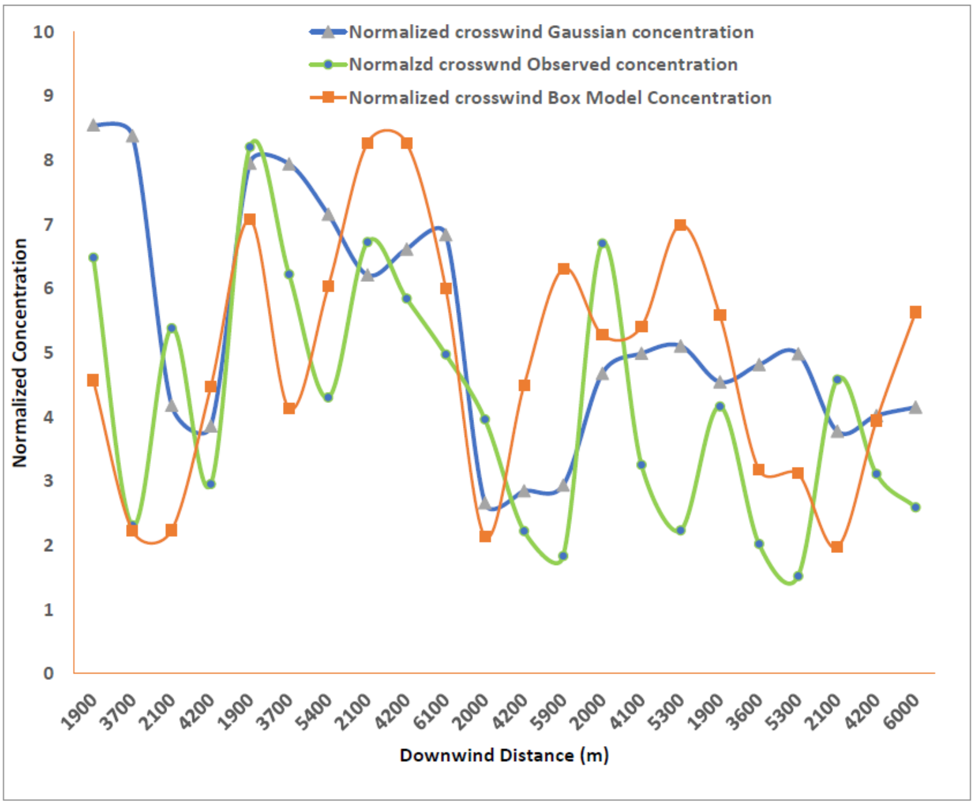

Fig. 4 – Shows the data of Normalized crosswind Gaussian, Box model and Observation concentrations (10^{-4} sm^{-2}).

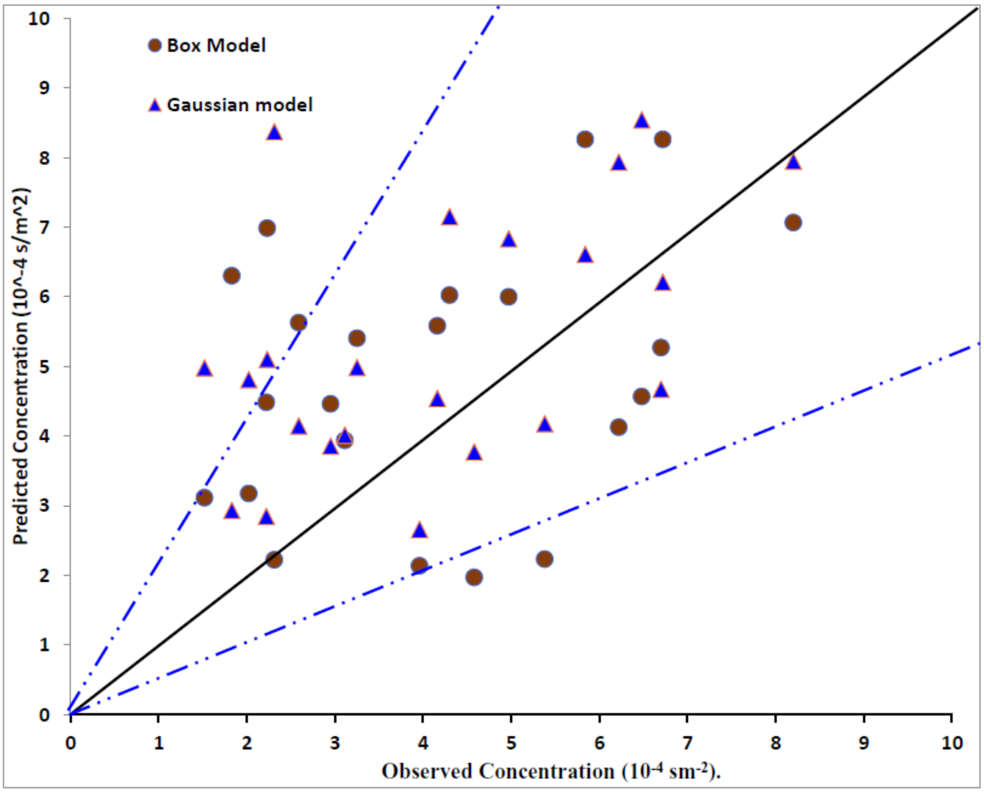

Fig. 5 – The achieving between Normalized crosswind Gaussians, Box model concentrations with normalized crosswind observed concentration.

Table 6

Comparison between Normalized crosswind Gaussian, and Box concentration models and normalized crosswind observed concentrations in Copenhagen, Denmark in neutral and unstable conditions

	NMSE	FB	COR	FAC2
Gauss. Eqns. (6,7)	0.21	–0.**25**	0.**52**	1.50
Box model Eq. (12)	0.**26**	–**0.16**	0.33	1.**40**
Irwin (Copenhagen)	0.85	–0.11	–0.11	1.44
Briggs (Copenhagen)	0.68	0.19	0.08	1.05

From Table 6, one can see that the integrated normalized lateral Gaussian concentration model is better than other mentioned models such as Box, Irwin and Briggs normalized crosswind concentration models. The present parameterization model played a great role in a good agreement of predictions with observations. This results of NMSE and FB are near from zero and COR and FAC2 are near from one in Gaussian than Box, Irwin and Briggs models.

CONCLUSION

The Gaussian plume model in three dimensions is better than the Box model because of the points of Gaussian model is nearer from the experiments data of I^{135} than the Box model. Also, Gaussian plume model in two dimensions located at factor of two than Box, Irwin, and Briggs models.

One finds that from the statistics that the integrated normalized lateral Gaussian concentration is better than Box, Irwin and Briggs models.

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