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STUDY THE EFFECT OF WIND VARIANCE, EULERIAN AND THE LAGRANGIAN INTEGRAL TIME SCALES ON THE DISPERSION PARAMETERS

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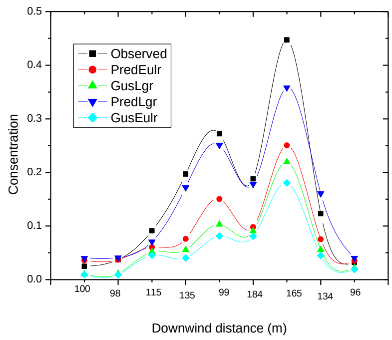
The simplest meaningful statistical measures of dispersion, we can compute the mean-square parameters displacement which must be increasing functions of time. This is in contrast to the variances of Lagrangian velocity fluctuations which must be independent of time in a stationary and homogeneous field of turbulence and equal to the corresponding Eulerian velocity variances. For small diffusion time, the mean-square particle displacement increases in proportion to square of diffusion time t^2". For large diffusion time, the mean-square diffusion eventually becomes proportional to Lagrangian integral time scale T_{iL} and diffusion time t. Considering these dispersion parameters in Gaussian diffusion model in three dimensions. These Gaussian and analytical concentrations results are compared with experimental data of Iodine-135 which collected in a convective boundary layer from Inshas which located at Egyptian Atomic Energy Authority.



INTRODUCTION

The statistical approach when applied to molecular or Brownian diffusion,[1,2] also leads to the same results as given by the phenomenological theory and verified by molecular diffusion experiments in laminar flows. The new approach does provide considerably more physical insight into the diffusion phenomenon and some useful information on diffusivities associated with the Brownian diffusion will not be covered here, the reader may refer to the original references given above or the more recent reviews.[3,4]

One-dimensional diffusion of tagged fluid particles released from the same point in a relatively simple, stationary, and homogeneous turbulent flow with zero or uniform mean velocity. The spatial homogeneity of turbulence is implied at least in the direction of diffusion, if not in all directions because it may not have any mean shear or unstable stratification in order to generate and maintain turbulence. The effect of any uniform mean motion is simply to transport particles at a fixed speed in the direction of flow. Let us consider the Lagrangian motion of a fluid particle initially located or released at the origin in the above frame of reference. The lateral and vertical dispersion parameters, respectively σy and σz, represent the turbulent parameterization in this approach, once they contain the physical ingredients that describe the dispersion process and, consequently. The values of σ_y and σ_z as

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functions of distance for use with his suggested stability categories. The modified values of σ_y and σ_z for use with the original Pasquill stability categories. The combination of Pasquill and Gifford parameters is called P-G scheme. In this scheme σ_y and σ_z are obtained from graphs as a function of downwind distance, *x*, for each stability class.

In this work, the mean-square parameters displacement, the variances of Lagrangian and Eulerian velocity fluctuations are calculated. Also, for large diffusion time, the mean-square diffusion eventually becomes proportional to Lagrangian integral time scale T_{iL} and diffusion time *t*. is calculated. The Gaussian and analytical concentrations results are compared with experimental data of Iodine-135 which collected in a convective boundary layer at Egyptian Atomic Energy Authority.

TECHNICAL METHOD

Although the Lagrangian motion is three dimensional, we can focus on particle movements in one direction (say, in the *Y* direction) only. The particle displacement *Y* from its initial position at the origin is only a function of time *t* after release. This is related to the Lagrangian turbulent velocity $V(\acute{t})$ as:

$$\acute{V}(t) = \frac{dY(t)}{dt}$$
$$Y(t) = \int_0^t \acute{V}(\acute{t})d\acute{t} \qquad (1)$$

The simplest meaningful statistical measures of dispersion, we can compute the mean square particle displacement $\overline{X^2}(t)$, $\overline{Y^2}(t)$ and so on which following the above reasoning must be increasing functions of time. This is the variances of Lagrangian velocity fluctuations, which must be independent time in a stationary and homogeneous field of turbulence and equal to the Eulerian velocity variances. The rate of change of the mean-square particle displacement $\overline{Y^2}(t)$ with time can be related to the statistics of turbulence, using Eq. (1) and Rynold's averaging rules as:

$$\frac{d\overline{Y^2}}{dt} = 2\bar{Y}(t)\frac{d\bar{Y}}{dt} = 2\left[\int_0^t \acute{V}(\acute{t})d\acute{t}\right]\acute{V}(\acute{t})$$
$$= 2\overline{\int_0^t \acute{V}(t)\,\acute{V}(\acute{t})d\acute{t}} \qquad (2)$$

In a stationary field of turbulence, the average of the velocity product in Eq. (2) will depend on the time difference $\xi = \acute{t} - t$ only and may be replaced by $\overline{\acute{V}^2}R_L(\xi)$, so that:

$$\frac{d\overline{Y^2}}{dt} = 2\overline{\acute{v}^2}\int_0^t R_L\,(\xi)d\xi \qquad (3)$$

where $R_L(\xi)$ is the autocorrelation coefficient of Lagrangian velocity.

The fundamental results the covariance between particle velocity and displacement was obtained as follows:

$$\overline{\acute{V}(t)Y(t)} = \frac{1}{2}\frac{d\bar{Y}^2}{dt} = \overline{\acute{v}^2}\int_0^t R_L\,(\xi)d\xi \qquad (4)$$

Integrating of Eq. (3) yields an alternative "Law of diffusion"

$$\overline{Y^2}(t) = 2\overline{\acute{v}^2}\int_0^t\int_0^{\acute{t}} R_L\,(\xi)d\xi d\acute{t} \qquad (5)$$

Similar expression can be written for $\overline{X^2}(t)$ and $\overline{Z^2}$. Eq. (5) was called "rather remarkable" because it reduces the problem of diffusion in a simplified type of turbulent motion to the consideration of a single quantity namely, the correlation coefficient between the velocity of a particle at one instant and that a time ξ later.

For small diffusion time:$t << T_{il}, R_L(\xi)$ does not differ appreciably from unity, so that Eqns. (3) and (5) can be approximated to:

$$\frac{d\overline{Y^2}}{dt} = 2t\overline{\acute{v}^2} \qquad (6)$$

$$\overline{Y^2}(t) = \overline{\acute{v}^2}t^2 \qquad (7)$$

Indicating that, initially mean-square particle displacement increases in proportion t^2.

For large diffusion time: $t >> T_{iL}$., one would expect the integral in Eq. (3) to approach a constant value, equal to the Lagrangian integral time scale T_{iL}, so that Eqns. (3) and (5) may be approximated to:

$$\frac{d\overline{Y^2}}{dt} = 2T_{iL}\overline{\acute{v}^2} \qquad (8)$$

$$\overline{Y^2}(t) = 2\overline{\acute{v}^2}T_{il}t \qquad (9)$$

Indicating that the mean-square diffusion becomes proportional to *t*, the Equations can also be written in terms of the standard deviation of particle displacement or so called one-particle diffusion parameter $\sigma_y = \left(\overline{Y^2}\right)^{0.5}$, as

$$\sigma_y = \sigma_v t \text{ for t} << \text{TiL} \qquad (10)$$

$\sigma_y = \sqrt{2}\sigma_v (T_{iL} t)^{0.5}$ for $t >> T_{iL}$ (11) when $t >> T_{iL}$, the covariance between the particle displacement and its velocity $\overline{Y\acute{V}}$, becomes constant in spite of the fact that $\overline{Y^2}$ continually increase. It obviously follows from Eqns. (4) and (8)

$$\overline{Y\acute{V}} = \frac{1}{2}\frac{d\overline{Y^2}}{dt} = \overline{\acute{v}^2} T_{iL} \quad \text{at t} >> \text{TiL} \tag{12}$$

Which implies that particle displacement must always be positively correlated with particle velocity, but the correlation coefficient must decrease with increasing diffusion time and, hence, with increasing $\overline{Y^2}$ since the above covariance or $0.5\, d\overline{Y^2}/dt$, will be shown equivalent to eddy diffusivity K_y, so Eq. (12) can also be consider as an expression for eddy diffusivity as a product of the variance of velocity and the integral time scale.

The same exponential form for $R_L(\xi)$ was assumed in deriving the following general expression for the diffusion parameter.

$$\sigma_y = \sigma_v T_{iL}\left[\frac{2t}{T_{il}} - 2\left(1 - e^{-\frac{t}{T_{iL}}}\right)\right]^{1/2} \tag{13}$$

Note that the exponential form of the autocorrelation coefficient is as follows:

$$R_L(\xi) = \exp\left(-\frac{|\xi|}{T_{iL}}\right) \tag{14}$$

In alternative form of the autocorrelation function which satisfies the above constraint is

$$R_L(\xi) = \exp\left(-\frac{\Pi}{4}\frac{\xi^2}{T_{iL}^2}\right) \tag{15}$$

Which implies the following expression for the dispersion parameter

$$\sigma_y = \sigma_v T_{iL}\left[\frac{2t}{T_{il}}\operatorname{erf}\left(\frac{\sqrt{\pi}}{2}\frac{t}{T_{iL}}\right) + \frac{4}{\pi}\exp\left(\frac{-\pi}{4}\frac{t^2}{T_{iL}^2} - \frac{4}{\Pi}\right)\right]^{1/2} \tag{16}$$

A useful simplification of Taylor is theorem is obtained by performing the integration of Eq. (5) with respect to t' by parts using Leibnitz's rule

$$\overline{\sigma_y}^2(t) = 2\overline{\acute{v}^2}\int_0^t (t-\xi)\, R_L(\xi) d\xi \tag{17}$$

This result can also be written in terms of the normalized Lagrangian spectrum function $S_L(n)$ by using Fourier transfer relations for formula:

$$R_L(\xi) = \int_0^\infty S_L(n)\cos(2\pi n\xi)\,\mathrm{d}n$$

One gets:

$$\overline{\sigma_y}^2(t) = 2t^2\overline{\acute{v}^2}\int_0^\infty S_L(n)\frac{sin^2(\pi n t)}{(\pi n t)^2}\mathrm{d}n \tag{18}$$

Similar expressions can be obtained for $\overline{\sigma_x}^2(t)$ and $\overline{\sigma_z}^2(t)$, provided turbulence is homogeneous in those directions also, for a completely homogeneous field of turbulence. Three dimensional versions of these results were derived. For simplicity, we discuss here only one-dimensional relations, which describe dispersion in one direction at a time and are often used in applications to atmospheric dispersion modeling.

Following the simplifying notation of the effect of smoothing of time series, which can be expressed from Eq. (18) as:

$$\overline{\sigma_y}^2(t) = 2t^2\overline{\acute{V}^2} \tag{19}$$

where $\overline{\acute{V}^2}$ represents the variance of the smoothed Lagrangian velocity with smoothing interval of *t*. Eq. (19) can also be interpreted as a simple but general relationship for dispersion parameter σ_y, as a product of diffusion time "t" and root-mean square of particle velocity averaged over *t*[6] *i.e.* $\sigma_y = \overline{V'}t$. The above dispersion relations entirely in terms of fixed point (Eulerian) statistics was expressed[7]. The basis of their modification is the simple hypothesis that the forms of $R_L(\xi)$ and $R_E(\tau)$ are similar as follows:

$$R_L(\xi) = R_E(\tau) \quad \text{for } \xi = \beta\tau \tag{20}$$

where, $\beta = T_{iL}/T_{iE}$ is the ratio of Lagrangian and Eulerian integral scales. The assumption of similarity between Lagrangian and Eulerian autocorrelation coefficients, also, implies the similarity between their corresponding spectral functions as:

$$nS_L(n) = \beta n S_E(\beta n) \tag{21}$$

Which follows from the appropriate Fourier transform relations

$$S(n) = 4\int_0^\infty R(\tau)\cos(2\pi n\tau)\,\mathrm{d}\tau$$

$$R(\tau) = \int_0^\infty S(n)\cos(2\pi n\tau)\,\mathrm{d}n$$

Between autocorrelation and spectrum functions. Substituting from Eq. (21) into Eq. (18), one obtains:

$$\overline{\sigma_y}^2(t) = \overline{\acute{v}^2}t^2\int_0^\infty S_E(n)\frac{sin^2(\pi n\acute{t})}{(\pi n\acute{t})^2}\mathrm{d}n \tag{22}$$

where, $\acute{t} = t/\beta$ may be considered as a modified diffusion or travel time. Note that Eq. (22) is similar to Eq. (18) but involves the more easily measured spectrum of Eulerian velocity. For the reduced variance of a smoothed time series Eq. (22) can also be written as:

$$\overline{\sigma_y{}^2}(t) = 4\overline{v_t'^2}t^2T_E \qquad (23)$$

where $\acute{t} = t/\beta$ is now the appropriate averaging time or smoothing interval, $S_E(n) = 4T_E$. Although Eq. (23) is quite similar to Eq. (19), it involves the more easily obtainable variance of Eulerian velocity after appropriate smoothing.

Plume diffusion from continuous sources

A continuous line, area, or volume source can be considered as a superposition of an infinite number of continuous point sources. Thus, the continuous point source can be used as the fundamental source geometry for describing diffusion from other continuous sources. Thus, in the usual coordinate system with the origin at the source position and the x-axis along the mean flow direction, the mean particle position from which displacements are considered (x,0,0) with $\bar{x} = \bar{u}t$.

Using the above simple transformation ($t = x/\bar{u}$) between the travel time and the average travel distance from the source, the dispersion relations of the previous section can be expressed in terms of the average travel distance from the source. For example, Eqns. (10) and (11) can be written as:

$$\sigma_y = \sigma_v t = \frac{\sigma_v}{u}x = i_v x \qquad \text{for x << Ly} \quad (24)$$

$$\sigma_y = \sqrt{2}\sigma_v(T_{iL}t)^{1/2} = \sqrt{2}\sigma_v\left(\frac{L_y x}{u^2}\right)^{1/2} = \sqrt{2}\frac{\sigma_v}{u}\left(L_y x\right)^{1/2} = \sqrt{2}i_v\left(L_y x\right)^{1/2} \qquad (25)$$

for x >> Ly

where, i_v is the turbulence intensity and $L_y = \bar{u}T_{iL}$ is a large-eddy length scale corresponding to the Lagrangian integral time scale. Similarly, the more general Hay-Pasquill relation (23) can be expressed as:

$$\sigma_y = \mathrm{i}_v' x \qquad (26)$$

where, i_v' is the reduced turbulence intensity obtained after smoothing the turbulence velocity signal with smoothing interval of $\acute{t} = x/\bar{u}\beta$. Note that i_v' is expected to be decreasing function of *x*, with its maximum value i_v near the source and becoming proportional to $x^{-1/2}$ far from the source.

Effects of finite sampling and release times

The effect of finite sampling duration on spectrum and variance of stationary variable (time series) has been discussed before. One can easily write the finite-sampling analogs of the statistical theory relations for example, from Eq. (5) and Eq. (18) respectively

$$\overline{\sigma_{y_T}^2} = 2\overline{\acute{v}_T^2}\int_0^t\int_0^t R_L(\xi)\mathrm{d}\xi\mathrm{d}\acute{t} \qquad (27)$$

$$\overline{\sigma_{y_T}^2} = \overline{\acute{v}_T^2}t^2\int_0^t S_L(n)\frac{Sin^2(\pi n t)}{(\pi n t)^2}dn = \overline{V_{T,t}^2}t^2 \qquad (28)$$

Where, the first subscript refers to the finite sampling duration and the second subscript indicates the finite smoothing or averaging interval.

The Hay-Pasquill approach described earlier yields the following dispersion relation entirely in terms of the fixed-point Eulerian velocity:

$$\overline{\sigma_{y_T}^2} = \overline{\acute{v}^2}t^2\int_0^\infty S_E(n)\frac{Sin^2(\pi n\acute{t})}{(\pi n\acute{t})^2}\left[1 - \frac{Sin^2(\pi nT)}{(\pi nT)^2}\right]\mathrm{d}n \qquad (29)$$

Which can be written as:

$$\overline{\sigma_{y_T}^2} = \overline{\acute{v}_{T,\acute{t}}^2}t^2 \qquad (30)$$

where, $\acute{t} = t/\beta$ represents the appropriate interval for smoothing or averaging the finite-duration time series $\acute{v}(t)$ before determining its variance.

Relation between concentration field and dispersion parameters

For a continuous point source, the average concentration at a point (*x*, *y*, *z*) is equivalent to the probability of finding a particle at (*x*, *y*, *z*) at any time. Considering contribution from all particles travel times, one can write

$$\bar{c}(x,y,z) = Q\int_0^\infty \rho(x,y,z;t)\mathrm{d}t \qquad (31)$$

where, Q is the emission rate of point source. With the Gaussian form of probability density functions of particle displacement in the y and z directions, one would expect from Eq. (31) the concentration distributions in y and z directions to be Gaussian also. It is easy to see that the width of two distributions must be identical dispersion parameters σ_y and $\sigma_z$also represent the standard deviations of concentration distributions across the plume in the y and z directions respectively, that is

$$\sigma_y = \left[\int_{-\infty}^{\infty}\bar{c}(x,y,z)y^2\,\mathrm{d}y/\bar{c}(x,y,z)\mathrm{d}y\right]^{1/2}$$

$$\sigma_z = \left[\int_{-\infty}^{\infty} \bar{c}(x,y,z)\, z^2 dz / \bar{c}(x,y,z) dz\right]^{1/2} \quad (32)$$

The process of diffusion by continuous movements implies an apparent turbulent diffusivity. This work has been examined in detail.[5,3] Starting from Gaussian expression for concentration distribution in a slender plume, resulting from a continuous point source in an infinite, homogeneous medium that is:

$$\bar{c}(x,y,z) = \frac{Q}{2\pi \bar{U} \sigma_y \sigma_z} \exp\left[\frac{-y^2}{2\sigma_y^2} - \frac{(z+H)^2}{2\sigma_z^2} - \frac{(z-H)^2}{2\sigma_z^2}\right] \quad (33)$$

where, H is the effective height, $H = h_s + 3\frac{w}{U}D$; w is the exist velocity; D is the diameter of the stack and U is the wind speed at stack height. One can show that the advective transport of material by mean flow is given by

$$\bar{u}\frac{\partial c}{\partial x} = \frac{\bar{u}}{2}\frac{d\sigma_y^2}{dx}\frac{\partial^2 c}{\partial y^2} + \frac{\bar{u}}{2}\frac{d\sigma_z^2}{dx}\frac{\partial^2 c}{\partial \sigma_z^2} \quad (34)$$

Eq. (34) can be recognized as being similar to the approximate diffusion equation

$$\bar{u}\frac{\partial c}{\partial x} = K_y \frac{\partial^2 c}{\partial y^2} + K_z \frac{\partial^2 c}{\partial \sigma_z^2} \quad (35)$$

In which diffusion in the direction of mean flow has been ignored. In fact, the two equations would be identical if eddy diffusivities were expressed in terms of dispersion parameters as:

$$K_y = \frac{u}{2}\frac{d\sigma_y^2}{dx} = \frac{1}{2}\frac{d\sigma_y^2}{dt}$$

$$K_z = \frac{u}{2}\frac{d\sigma_z^2}{dx} = \frac{1}{2}\frac{d\sigma_z^2}{dt} \quad (36)$$

In which, we have used the usual transformation between travel time and distance from the source. From Eq. (3), these apparent eddy diffusivities can also be expressed as:

$$K_y = \acute{v}^2 \int_0^t R_{Lv}(\xi)\, d\xi$$

$$K_z = \acute{w}^2 \int_0^t R_{Lw}(\xi)\, d\xi \quad (37)$$

Similar expression for eddy diffusivities as given in Eq. (36) with the addition of $K_x = \frac{1}{2}\frac{d\sigma_x^2}{dt}$, can be derived for puff diffusion,[3] if one requires that the Gaussian puff formula for an instantaneous point source in an infinite medium satisfy the mean diffusion equation.

Taylor's theorem gives the following useful relations between eddy diffusivities and other turbulence statistics:

$$K_x = \acute{u}^2 T_{Lu};\ K_y = \acute{v}^2 T_{Lv};\ K_z = \acute{w}^2 T_{Lw} \quad (38)$$

which are applicable only in the limit of large diffusion time $t \gg T_{iL}$.

The scaling factor for the i-velocity component (u, v, w) defined as the ratio between Lagrangian and Eulerian time scales from[8] as follows:

$$B_i = \frac{T_{L_i}}{T_{E_i}} = \frac{0.55U}{\sigma_i} \quad (39)$$

The Lagrangian time scales T_{L_i} can be written in the form:[9]

$$T_{L_v} = \frac{0.3h}{\psi^{1/3} w_*} \quad (40)$$

$$T_{L_w} = 0.33\frac{h}{\psi^{1/3} w_*}\left[1 - \exp\left(-4\frac{z}{h}\right) - 0.0003\exp\left(-8\frac{z}{h}\right)\right]^{2/3} \quad (41)$$

where, w_* is the convective velocity scale, ψ is the nondimensional molecular dissipation rate function and h is the convective boundary layer (CBL). The Lagrangian variance of particle position is given by Eq. (9):

$$\bar{X}_i^2 = 2u_i^2 t T_{iL} \quad (42)$$

where, $t = x/U$ is the diffusion travel time, U is the horizontal mean wind speed and $u_i^2 = \sigma_i^2$ are the velocity variances.

In the Convective Boundary Layer (CBL) the velocity variances can be obtained[10] from the w_* so that $\sigma_w^2 = 0.42 w_*^2$ and $\sigma_v^2 = 0.36 w_*^2$.

Then, the vertical dispersion parameter in Lagrangian case can be written from Eq. (42) as follows:

$$\sigma_z^2 = 0.27\frac{h^2 X}{\psi^{1/3}}\left[1 - \exp\left(-4\frac{z}{h}\right) - 0.0003\exp\left(-8\frac{z}{h}\right)\right]^{2/3} \quad (43)$$

where, $X = (w_* x)/(Uh)$ is the nondimensional distance with[11] $\psi = 1.5 - 1.2(z/h)^{1/3}$. Also, the lateral dispersion parameter in Lagrangian case can be written from Eq. (42) as follows:

$$\sigma_y^2 = 0.22\frac{h^2}{\psi^{1/3}}X \quad (44)$$

where, $\psi = \frac{\varepsilon h}{w_*^3}$, $\varepsilon = 0.65$, Eqns. (43) and (44) are used in Eq. (33) at y = 0 to get the Gaussian plume model at centerline $\bar{c}(x,0,z)$as follows:

$$\bar{c}(x,0,z) = \frac{Q}{2\pi\bar{U}\sigma_y\,\sigma_z}\exp\left[-\frac{(z+H)^2}{2\sigma_z^2} - \frac{(z-H)^2}{2\sigma_z^2}\right]\exp\left(-\frac{\nu x}{U}\right) \quad (45)$$

where, $e^{-\frac{\nu x}{u}}$is the radioactive decay for the specified nuclide, $\nu = 2.9 * 10^{-5}s^{-1}$.

Also, from Eq. (39), The Eulerian time scales T_{E_i} can be written in the form"

$$T_{E_v} = T_{L_v}\sigma_v/0.55U \quad (46)$$

Substituting from Eq. (40) in Eq. (46), one gets:

$$T_{E_v} = 0.38h^{2/3}w_*/U \quad (47)$$

Also, substituting from Eq. (41) in Eq. (46), to get the vertical Eulerian time scales T_{E_W}as follows:

$$T_{E_W} = 0.39\frac{h}{U\psi^{1/3}}\left[1-\exp\left(-4\frac{z}{h}\right) - 0.0003\exp\left(-8\frac{z}{h}\right)\right]^{2/3} \quad (48)$$

The Eulerian variance of particle position is given by Eq. (23) as follows:

$$\bar{X}_i^2 = 4u_i^2t^2T_{E_i} \quad (49)$$

Then, the vertical dispersion parameter in Eulerian case can be written from Eq. (49) as follows:

$$\sigma_z^2 = 0.66\frac{h^3X^2}{U\psi^{1/3}}\left[1-\exp\left(-4\frac{z}{h}\right) - 0.0003\exp\left(-8\frac{z}{h}\right)\right]^{2/3} \quad (50)$$

Also, the lateral dispersion parameter in Eulerian case can be written from Eq. (49) as follows:

$$\sigma_y^2 = 0.55\frac{w_*^3x^2h^{2/3}}{U^3} \quad (51)$$

Substituting from Eqns. (50) and (51) are used in Eq. (45) at y = 0 to get the Gaussian plume model at centerline $\bar{c}(x,0,z)$

Mathematical model. The Diffusion equation in three dimensions is

$$u\frac{\partial C(x,y,z)}{\partial x} = \frac{\partial}{\partial y}\left(k_y\frac{\partial C(x,y,z)}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z\frac{\partial C(x,y,z)}{\partial z}\right) \quad (52)$$

where, $C(x,y,z)$ is the concentration of pollutants (g/m^3) or (Bq/m^3), k_y and k_z are the eddy diffusivities in crosswind and vertical direction respectively, $u$is the wind speed (m/s), x is downwind distance (m).

By taking crosswind integration with respect to y from -∞ to ∞, one gets diffusion equation in two dimensions as follows:

$$u\frac{\partial C_y(x,z)}{\partial x} = \frac{\partial}{\partial z}\left(k_z\frac{\partial C_y(x,z)}{\partial z}\right) \quad (53)$$

where,$C_y(x,z)$ is the crosswind integrated concentration of pollutants. Eq. (2) is solved under the boundary conditions as follows:

(a) The condition of null flux is applied on the ground surface and at the mixing height.

$$k_z\frac{\partial C}{\partial z} = 0 \quad \text{at} \quad z = 0, h \quad (53a)$$

(b) The mass continuity is used.

$$uC_y(0,z) = Q\delta(z-h) \text{ at } x = 0 \quad (53b)$$

where, h is the heightof the atmospheric boundary layer (ABL)(m), Q is the emission rate (g/s) or (Bq), and δ is a Dirac delta function.

(c) The crosswind integrated concentration tends to zero as z tends to ∞

$$C_y(x,z) \to 0 \quad \text{as } z \to \infty \quad (53c)$$

(d) The crosswind integrated concentration vanishes at the mixing height

$$C_y(x,z) = 0 \quad \text{at } z = h \quad (53d)$$

Now the advection-diffusion equation in two dimensions Eq. (53) will be solved by second mathematical model, considering the height of ABL (h) is discretized into N sub-interval layers such that within each interval, k_z and u are taken as average values. Then the solution of Eq. (53) is reduced to the solutions of N equations of the following type

$$u_i\frac{\partial C_y(x,z)}{\partial x} = k_i\frac{\partial^2 C_y(x,z)}{\partial z^2} \quad (54)$$

where,

$$k_i = \frac{1}{z_{i+1} - z_i} \int_{z_n}^{z_{i+1}} k_i(z)\,\mathrm{d}z$$

$$u_i = \frac{1}{z_{i+1} - z_i} \int_{z_i}^{z_{i+1}} u_i(z)\,\mathrm{d}z$$

for, $z_i \leq z \leq z_{i+1}$, $i = 1 : N$

then the concentration in three dimensions is taken from[12]

$$C(x, y, z, h_s) = \frac{Q}{2\sqrt{2x}\,\pi\,\sigma_y u_i} e^{-\frac{(z-h)^2 u_i}{4k_i\,x} - \frac{y^2}{2\sigma_y^2} - \frac{\beta x}{u}} \quad (55)$$

where, u_i and k_i are taken from two equations (3) and (4) respectively. σ_y is the standard deviation in y direction and $e^{-\frac{\beta x}{u}}$ is the radioactive decay for the specified nuclide, β is radioactive coefficient.

RESULTS AND DISCUSSION

The air samples of observed isotope concentrations of I^{135} in unstable condition were collecting from the First Research Reactor at Inshas located at Egyptian Atomic Energy Authority. The experiments were observed from a stack height is 43 m with a roughness length of 0.6 m. The meteorological data of I^{135} are considered from[13] are shown in Table 1. The Gaussian plume model concentrations are taken from Eq. (45) and the proposed concentrations by Eq. (55) below the plume centerline are given in Table 2. Where the crosswind and vertical dispersion parameters were calculated from Eqns. (43) and (44) for Lagrangian case and Eqns. (50) and (51) for Eulerian case. One finds that the proposed model is well agreement with observed concentrations and the proposed data inside a factor of two in unstable condition as shown in Figs 1, 2.

Table 1

Meteorological data of the nine convective test runs at Inshas, Egypt in March and May 2006

Run no.	Working hours of the source	Release rate (Bq)	Wind speed U10 (m s^{-1})	Wind direction (deg)	U43 (m s^{-1})	W* (ms^{-1})	P-G stability class	h (m)	Vertical distance (m)
1	48	1028571	4	301.1	4.97829	2.27	A	600.85	5
2	49	1050000	4	278.7	4.97829	3.05	A	801.13	10
3	1.5	42857.14	6	190.2	7.46744	1.61	B	973	5
4	22	471428.6	4	197.9	5.35493	1.23	C	888	5
5	23	492857.1	4	181.5	4.97829	0.958	A	921	2
6	24	514285.7	4	347.3	5.76006	1.3	D	443	8.0
7	28	1007143	4	330.8	5.35493	1.51	C	1271	7.5
8	48.7	1043571	4	187.6	5.35493	1.64	C	1842	7.5
9	48.25	1033929	4	141.7	4.97829	2.1	A	1642	5.0

Table 2

Meteorological parameters and concentrations measured during the Inshas, Egypt experiment in unstable condition.

Run	Stability class	Distance (m)	Q (Bq)	Observed C(Bq/m^3)	GaussLgr C(Bq/m^3)	PredLgr C(Bq/m^3)	GaussEur C(Bq/m^3)	PredEur C(Bq/m^3)
1	A	100	1028571	0.025	0.01018	0.04027	0.00948	0.03599
2	A	98	1050000	0.037	0.01131	0.04112	0.00925	0.03711
3	B	115	42857.14	0.091	0.05172	0.0708	0.04567	0.06057
4	C	135	471428.6	0.197	0.05563	0.17212	0.0406	0.07595
5	A	99	492857.1	0.272	0.10296	0.2509	0.08119	0.1505
6	D	184	514285.7	0.188	0.0901	0.17823	0.08107	0.0981
7	C	165	1007143	0.447	0.21962	0.35818	0.18054	0.2504
8	C	134	1043571	0.123	0.05583	0.16083	0.04512	0.0753
9	A	96	1033929	0.032	0.0221	0.04038	0.01916	0.03511

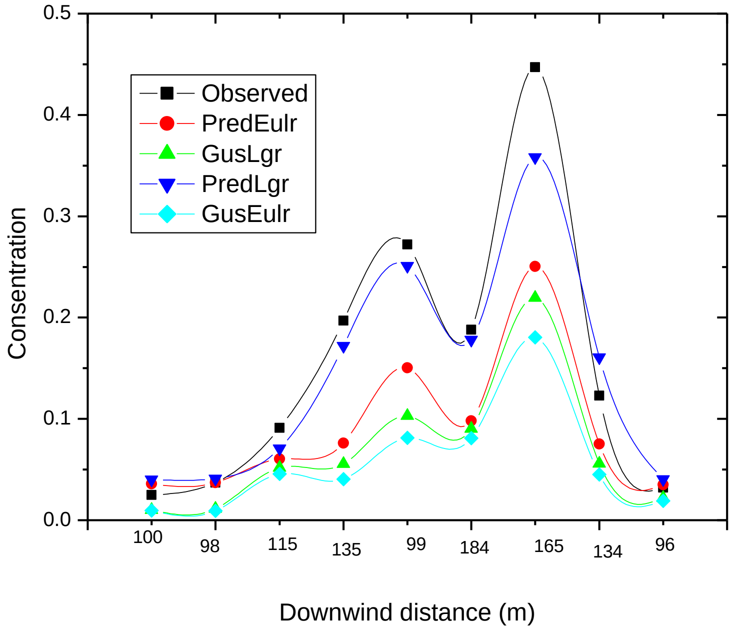

Fig. 1 – the variations of observed, Gaussian and proposed concentrations models via downwind distance (m) in Lagrangian and Eulerian forms.

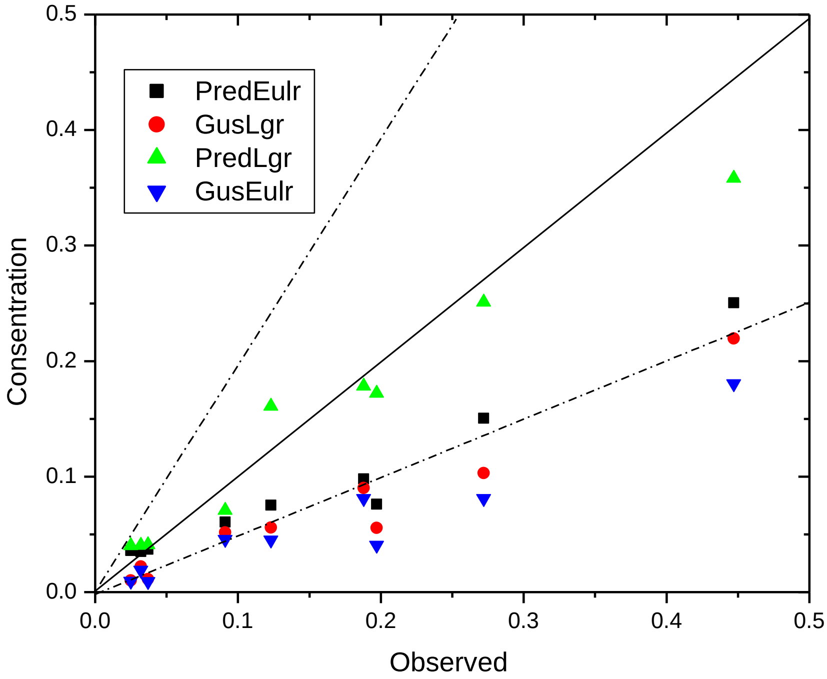

Fig. 2 – the variations of Gaussian and proposed models via observed concentrations in Lagrangian and Eulerian forms.

Figure 1 shows that the proposed concentrations in lagrangian are nearer the observed concentrations data of I^{135} than the proposed concentrations in Eulerian form, the Gaussian in Lagranian and the gaussian in Eulerian form. All the Gaussian and the proposed using Lagrangian, and the proposed using Eulerian lay inside a factor of two but the Gaussian in Eulerian lay inside a factor of three as shown in Fig. 2.

Statistical Technique

Comparing between Gaussian, proposed and observed concentrations is introduced[14]. where, NMSE is the Normalized Mean Square Error, FB is the Fraction Bias, COR is the Correlation coefficient and FAC2 is the Factor of Two.

Table 3

Comparison between Gaussian, proposed and observed concentrations using Lagrangian and Eulerianin unstable condition

	NMSE	FB	COR	FAC2
Gauss. Lagranian	0.62	0.78	0.97	0.44
Proposed lagrangian	0.009	0.07	0.98	0.93
Proposed Eulerian	0.3	0.53	0.98	0.58
Gaussian Eulerian	0.94	0.94	0.96	0.36

One can see easily from Table (3), the entire proposed model in Lagrangian and Eulerian lay inside a factor of two with observed concentration data of I^{135} But Gaussian in Lagrangian and Eulerian lay outside a factor of two. Also, the statistical shows that the proposed in Lagrangian or Eulerain are better than Gaussian in Lagrangian and Eulerian. Also, the Gaussian in Lagrangian model is better than Gaussian in Eulerian.

CONCLUSIONS

The proposed concentrations in lagrangian are nearer the observed concentrations data of I^{135} than the proposed concentrations in Eulerian form, the Gaussian in Lagrangian and in Eulerian form. All the Gaussian and the proposed using Lagrangian, and the proposed using Eulerian lay inside a factor of two but the Gaussian in Eulerian lay inside a factor of three.

One can see easily, the entire proposed model in Lagrangian and Eulerian lay inside a factor of two with observed concentration data of I^{135} But Gaussian in Lagrangian and Eulerian lay outside a factor of two. Also, the statistical shows that the proposed in Lagrangian or Eulerain are better than Gaussian in Lagrangian and Eulerian. Also, the Gaussian in Lagrangian model is better than Gaussian in Eulerian.

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