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THE EFFECT OF LOGARITHMIC AND POWER LAW OF WIND SPEED ON GROUND LEVEL CONCENTRATION IN AREA SOURCES

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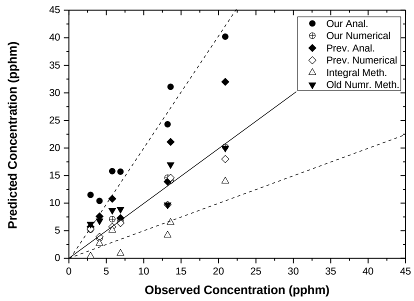
Two-dimensional diffusion equation is solved analytically and numerical solutions by separation and finite difference methods respectively. The wind velocity consists of the power and logarithmic laws to get the theoretical and numerical concentrations. The proposed and numerical models are used to compare with measured concentrations of Sulphur dioxide concentration distribution over two hours period in Nashville, Tennessee USA and also, the before work which was made before.
Our proposed concentration is found to be larger than the observations, our previous methods of integral and numerical methods.



INTRODUCTION

A new approach for solving dispersion of pollutant in the planetary boundary layer in the planetary boundary layer. was used by.[1,2]

Mathematical technique considering the eddy diffusivity and wind speed were constant was estimated.[2] The solution of diffusion equation under dry deposition to the ground was obtained.[3] Two- and three-time dependent advection-diffusion equation in different stability conditions was solved.[4]

Theoretical diffusion model for area source by an integral method was studied.[5] The numerical method in simulating the change of Sulphur dioxide through the air was calculated.[6] A review of current issues in air pollution modeling and simulation was studied.[7]

In this paper, we calculate a simple solved of diffusion equation in the steady state which contains of the logarithmic and power law of wind velocity using integral transform and finite difference methods. These methods have been giving solutions which larger than observed, previous methods of integral and numerical methods concentration of Sulphur dioxide.

METHOD

The concentration c(x, z) of pollutant at a point (x, z) in steady state of the advection-diffusion

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equation under several hypothesis is obtained in the form as follows:

$$u(z)\frac{\partial C}{\partial x} = \frac{\partial}{\partial x}\left[K(z)\frac{\partial C}{\partial z}\right] \quad (1)$$

where u is the average wind velocity and K(z) is the vertical turbulent in the vertical height. One lets that the plume is emitted as steady flux at the ground "z = 0" as follows:

$$K(z)\frac{\partial C}{\partial z} = -Q \quad \text{at } z = 0 \quad (2)$$

where, Q is the amount of release from the stack which is constant with downwind distance "x". h(x) is a mixing height, which is as function in "x", and let there is no concentration over h(x) as follows:

$$C(x,z) = 0 \quad at\ z \geq h(x) \quad (3)$$

Also, there is no flux above the mixing height as follows:

$$\frac{\partial C}{\partial z} = 0 \quad at\ z \geq h(x) \quad (4)$$

Let at the top of the planetary boundary layer, the concentration varies smoothly in the x-direction as follows:

$$\frac{\partial C}{\partial x} = 0 \quad at\ z = h(x) \quad (5)$$

Under the above condition, one gets:

$$\frac{\partial^2 C}{\partial Z^2} = 0 \quad at\ z = h(x) \quad (6)$$

Taking the wind velocity consists of power and logarithmic laws and the vertical diffusion coefficient may be written[9] as follows:

$$u(z) = \frac{u_*}{k}\left[Ln\frac{z}{z_o}\right] + u_1\left(\frac{z}{z_1}\right)^m ; K(z) = k_0 + k_1\left(\frac{z}{z_1}\right)^n \quad (7)$$

At the ground, one finds that $K = k_0$, *i.e.*, the diffusion shouldn't equal zero at the point of release, u_* is the friction velocity, k is Von-Karman constant equals 0.4, z_0 is the roughness height, u_1 is the wind velocity at reference height 10 m and k_1 is the vertical turbulent at reference height 10 m.

Let Eq. (1) has solution as a polynomial of third degree as follows:

$$C(x,z) = f(x)\left(1 - \frac{z}{h}\right)^3 \quad (8)$$

Using Eq. (2) one gets:

$$f(x) = \frac{h(x)Q}{3k} \quad (9)$$

where, $h(x)$ is obtained by integrating Eq. (1) to "z" from "0" to "h", using Eq. (2) and integrating it over x, assuming that $f(x_0) = 0$ i.e., $h(x_0) = 0$ and $u_* = 0.4$ in neutral case, one gets from[5,9] as follows:

$$h(x) = exp\left[0.5\ Lambert\ W\left(-1.1164\frac{(x-x_0}{z_0^2} + 2.0833\right)\right]z_0 + \left[\frac{3z_1^m k}{u_{1\ B(m+1,4)}}(x - x_0)\right]^{1/(m+2)} \quad (10)$$

where, $LambertW(x)e^{LambertW(x)} = x$

Then Eq. (9) becomes:

$$f(x) = \left\{\frac{Q}{3k}\left[exp\left[0.5\ Lambert\ W\left(-1.1164\frac{(x-x_0}{z_0^2} + 2.0833\right)\right]z_0 + \left[\frac{3z_1^m k}{u_{1\ B(m+1,4)}}(x - x_0)\right]^{1/(m+2)}\right]\right\} \quad (11)$$

But for an area source, the concentration at the surface is obtained from[5,9] as follows:

$$f(x) = \frac{1}{3k}\sum_{L=1}^{s}(Q_L - Q_{L-1})z_0\left\{\begin{array}{c} exp\left[0.5\ Lambert\ W\left(-1.1164\frac{(x-x_0}{z_0^2} + 2.0833\right)\right] \\ + \left[\frac{3z_1^m k}{u_{1\ B(m+1,4)}}(x - x_0)\right]^{1/(m+2)}\end{array}\right\} \quad (12)$$

For the numerical integration of Eq. (1), the Crank-Nicolson scheme has been applied yielding the finite difference form.[9,5]

$$\begin{aligned} C_{i+j} = \alpha\left[\left(\frac{1}{\alpha} + 2k_j + k_{j+1} - k_{j-1}\right)\right]C_{i+1,j} + \left(2k_j + k_{j+1} - k_{j-1}\right)C_{i+1,j+1} - \\ -\left(k_{j-1} - k_j\right)\left(\frac{2\Delta z Q}{k_0}\right) \quad at \quad j = 1, 2, \ldots, N \end{aligned} \quad (13)$$

where, $\alpha = \Delta x / 4u(\Delta z)^2$, the integral between the grid points i and i+1 is Δx, the vertical interval between the points j and j+1 is Δz, which are solved simultaneously.

An analytical and numerical methods is estimated using two Eqs. (12) and (13) to evaluate the concentration at the ground.

Dispersion of Sulphur dioxide

One applies the two Eqs. (12) and (13) to obtain the dispersion of SO_2 in the air of Nashville, Tennessee, USA over two – hour's period source inventory and meteorological data for the problem is taken from Randerson (1970). Measured values of SO_2 concentration at seven observed stations are used to compare with the values estimated by two Eqs. (12) and (13), other integral and numerical methods[9] and integral method[5] and old numerical method.[6]

Table 1

Observed and estimated Sulphur dioxide concentration in Nashville are given in pphm (part per hundred million) where 1pphm = $2.7\times10^{-5}g\cdot m^{-3}$ of So_2

Observation No.	Observation concentration	Our analytical model	Our Numerical model	Previous Analytical Wahab *et al.* (2009)	Previous numerical Wahab *et al.* (2009)	Previous Integral Method Lebedeff Hameed (1975)	Previous old numerical Randerson (1970)
19	5.8	15.8	10.8	7.1	5.7	8.7	5.1
48	13.6	31.1	21.1	14.1	14.6	17	6.5
52	2.9	11.5	5.7	5.2	5.3	6.2	0.4
56	6.9	15.7	7.3	6.8	6.4	8.9	0.9
60	20.9	40.2	32	20.2	18	20	14
80	13.2	24.3	13.9	14.6	9.7	9.7	4.2
90	4.1	10.4	7.6	3.6	3.9	6.8	2.7
Correlation coefficient		0.98	0.95	0.99	0.96	0.92	0.89

Table (1) shows that observed and proposed concentrations, first our models for theoretical and numerical methods using logarithmic and power laws of wind velocity, conduct work of integral and numerical models using logarithmic law of wind velocity[9] and previous work of integral and numerical methods using power law of wind velocity[5,6] respectively. The correlation coefficients of the new work, two conduct calculated methods before,[5] are good as the correlation coefficients of conduct work before.[5,6]

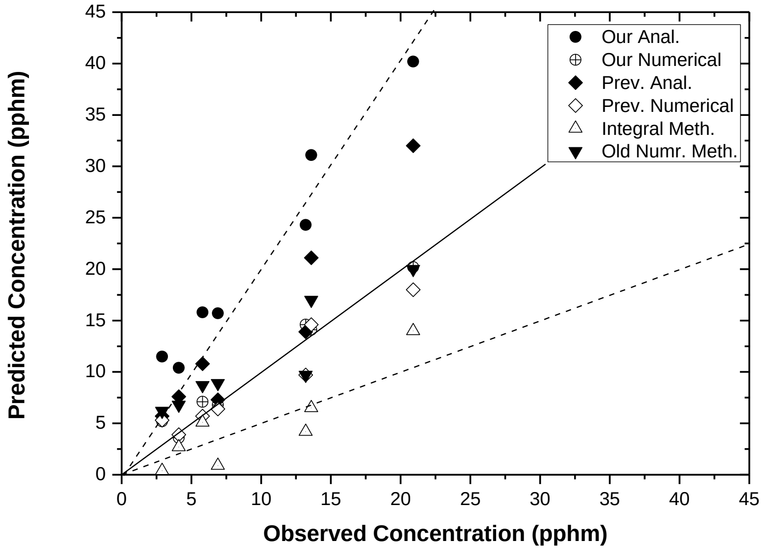

Fig. 1 – Variation between estimated and observed concentration of Sulphur dioxide at Nasville, Tennessee, where pphm is part per hundred million.

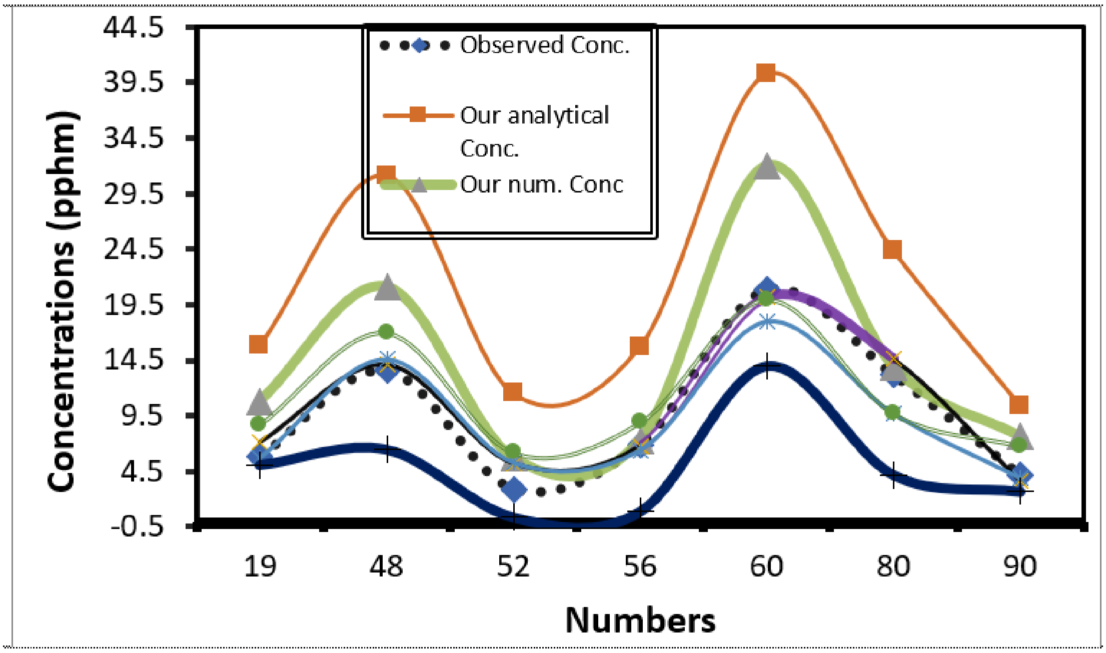

Fig. 2 – shows the variation of all analytical, numberical and observed concentrations (pphm) via experimental numbers.

Our analytical proposed model using Eq. (12) is located inside a factor of three but our numerical model, our conduct work (analytical and numerical),[9] previous numerical method[6] and most points of conduct analytical method[9] are located a factor of two but our analytical method is located a factor of three as shown in two Figs. 1, 2.

The datasets of estimated concentrations were applied to the following statistical indices.[10]

NMSE (normalized mean square error) = $\overline{(C_o - \bar{C}_p)}/C_o C_p$, Fa2 (Factor of two (%) = 0.5 ≤ $(C_{p/}C_o) \leq 2$, Cor (correlation coefficient) = $\overline{(C_o - \bar{C}_o)(C_p - \bar{C}_p)}/\sigma_o\sigma_p$, Fb (Fractional bias) = $(\bar{C}_o - \bar{C}_p)/0.5(\bar{C}_o + \bar{C}_p)$ and Fs (Fractional standard) deviation = $(\sigma_o - \sigma_p)/0.5(\sigma_o + \sigma_p)$, where subscript o and p tends to measured and proposed quantities, respectively and an over bar indicated an average.

Table 2

Explains the statistical technique for all work

Model	NMSE	Fb	Fs	Fa2	Cor
Our analytical model	0.76	–0.75	–0.522	2.21	0.98
Our numerical model	0.24	–0.37	–0.373	1.46	0.95
Previous analytical model*	0.01	–0.06	0.061	1.14	0.99
Previous numerical model*	0.05	0.06	0.198	1.05	0.96
Integral method**	0.07	–0.14	0.202	1.36	0.93
Old numerical method***	0.69	0.66	0.342	0.47	0.89

*[9], **[5] and *** [6].

CONCLUSIONS

Using the wind velocity consists of power and logarithmic laws to get the theoretical and numerical solution of advection-diffusion equation in two dimensions. Taking our proposed results, the conduct work,[9] conduct work of integral method[5] and Numerical method[6] to compare with the measured concentration of Sulphur dioxide.

Our analytical proposed model using Eq. (12) is located a factor of three but our numerical model, our conduct work (theoretical and numerical),[9] conduct numerical method[6] and most points of theoretical method is located a factor of two but our theoretical method is located a factor of three.

Our integral and numerical methods give larger results than the previous work of integral and numerical methods.[5,6,9]

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REFERENCES

1. Wortmann M., Vilhena, S., Twfulerce M., Moreira, D. M. and Buske D., *Atmos. Environ*., **2005,** *3a*, 2171–2178.
2. Essa K. S. M., Soad M., Etman M. and Embaby M., *Atmosphere Research*, **2007**, *84*, 337–344.
3. Kumar P. and Sharan M., *Aerosol and Air Quality Research*, **2016**, *16*, 1284–1293.
4. Essa K.S.M., A. N. Mina, H. S. Hamdy, F. Mubarak, A. M. Mosallem and A. A. Khalifa, *Interciencia journal*, **2019**, *44*, 4.
5. Lebedeff S. A. and Hamead S., *Atmos. Environ.*, **1975**, *9*, 333–338.
6. Randerson D., *Atmos. Environ.*, **1970**, *4*, 615–632.
7. Sportisse B., *Comput. Geosci*., **2007**, *11*, 159–181.
8. Hanna S. R., Briggs G. A. and Hosker R. P. Jr., “Handbook on atmospheric diffusion”, U.S. Department of Energy Report DOE / TIC-11223. Washington, DC, 1982.
9. Wahab M., K. S. M. Essa, Embaby M. and Sawsan E. M. Elsaid, Ground Level Concentrations on Area Sources, Mausam, 60, 4 (October 2009).
10. M. Z. Jacobson, “Fundamentals of atmospheric Modeling”, Cambridge University Press, 1999, p. 167.
11. Hanna S. R., *Atmos. Environ*., **1989**, *23*, 1385–1395.