



Dedicated to Professor Gheorghe Maria
on the occasion of his 70th anniversary

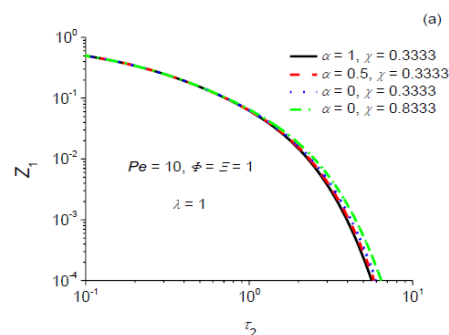
CONJUGATE HEAT TRANSFER BETWEEN A SPHERE AND A MICROPOLAR FLUID IN CREEPING FLOW

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The unsteady, conjugate, forced convection heat transfer from a rigid sphere to a micropolar fluid in creeping flow has been analysed. The heat balance equations were solved numerically. The influence of the non – Newtonian parameters of the micropolar fluid on the heat transfer mechanism and rate was analysed for different values of the physical properties ratios (thermal conductivity ratio and heat capacity ratio) and $Pe = 10, 100$.



INTRODUCTION

The momentum, heat and mass transfer between a sphere and a surrounding medium is one of the classical benchmark problems for the analysis of transfer phenomena. The forced convection heat / mass transfer between a sphere and a micropolar fluid flow is one of the particular cases of this problem. Micropolar fluids,^{1,2} represent fluids containing rigid, randomly oriented particles suspended in a viscous medium. The mathematical model of micropolar fluids is a theoretical generalization of the Navier-Stokes model. Ferrofluids, blood flows, liquid crystals, bubbly liquids, granular flows are some of the applications of the micropolar fluid.

Lien and Chen,³ Wang & Kleinstreuer⁴ and Juncu⁵ investigated forced convection heat or mass transfer from a spherical particle to a micropolar fluid flow. Lien and Chen³ and Wang and Kleinstreuer⁴ solved the steady external problem using the boundary layer approximation (for both flow and heat transfer). Two boundary conditions on the surface of the sphere were considered: constant temperature and constant heat flux. For the case of moderate Re numbers flow, the unsteady conjugate heat transfer was analysed numerically in.⁵

Much attention received free convection heat / mass transfer from a sphere to a micropolar fluid.^{6–11} The boundary layer approximation with the same boundary conditions on the surface of the sphere,

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i.e. constant temperature / concentration and constant heat / mass flux, were employed. The same mathematical models (boundary layer) and boundary conditions, were employed for the analysis of the mixed convection heat / mass transfer from a sphere to a micropolar fluid.^{12,13}

Problems related to the one analysed in this work are: (1) free convection of a micropolar fluid in a spherical annuli;¹⁴⁻¹⁷ (2) forced convection heat transfer from a circular cylinder to a micropolar fluid flow.¹⁸ For these problems, numerical solutions were obtained considering the general convection – diffusion equations. However, additional discussions concerning these topics are outside the aims of the present study.

The aim of the present work is to complete the analysis from⁵ with the creeping flow case. This

problem was not investigated until now. The present computations focus on the influence of the micropolar fluid parameters on the heat transfer mechanism and rate for different values of the physical properties ratios (thermal conductivity ratio and heat capacity ratio) and $Pe = 10, 100$.

MODEL EQUATIONS

Figure 1 shows the physical model for the present problem. The fluid is micropolar and the flow is laminar, steady, viscous, axisymmetric and incompressible. The physical properties are uniform, isotropic and constant. The effects of buoyancy, viscous dissipation, work done by pressure changes and radiation are neglected.

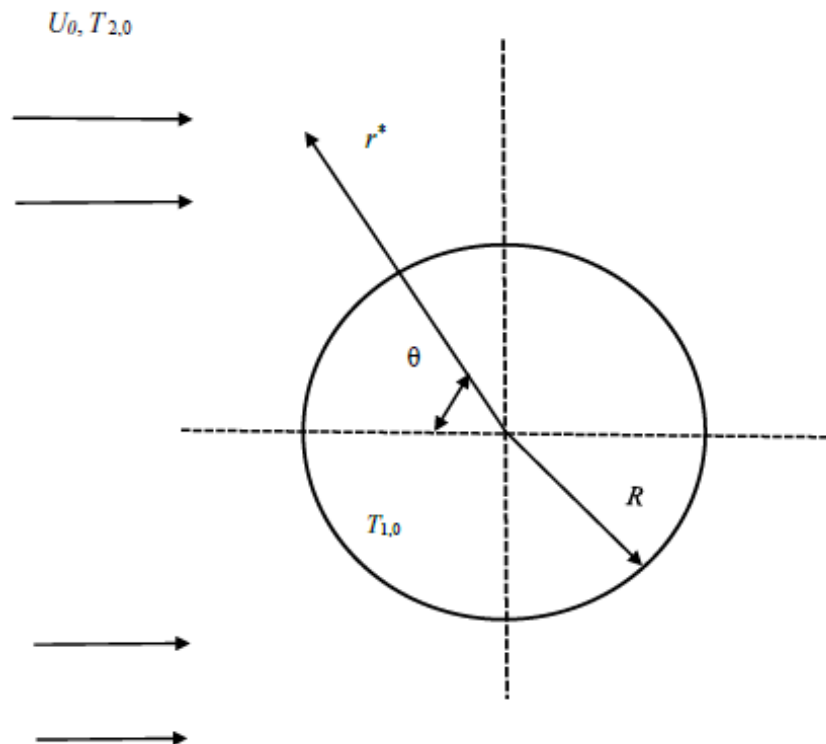


Fig. 1 – Schematic of the problem.

Considering also that during the heat transfer there is no phase change, the conjugate heat transfer is governed by the following dimensionless equations:

– for the interior of the sphere ($r < 1$), $i = 1$;

$$\frac{\partial Z_1}{\partial \tau_1} = \Delta Z_1 \quad (1a)$$

– for the exterior of the sphere ($r > 1$), $i = 2$;

$$\frac{\partial Z_2}{\partial \tau_2} + \frac{Pe}{2} \left(V_R \frac{\partial Z_2}{\partial r} + \frac{V_\theta}{r} \frac{\partial Z_2}{\partial \theta} \right) = \Delta Z_2 \quad (1b)$$

where

$$\Delta = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right), \quad Pe = \frac{U_0 d (\rho c_p)}{k}.$$

The boundary conditions are:

– axis of symmetry, $\theta = 0, \pi$

$$\frac{\partial Z_i}{\partial \theta} = 0, i = 1, 2 \quad (2a)$$

– surface of the sphere, $r = 1$

$$Z_1 = Z_2, \Phi \frac{\partial Z_1}{\partial r} = \frac{\partial Z_2}{\partial r}, \quad (2b)$$

– center of the sphere, $r = 0$
 $Z_1 = \text{finite}, \quad (2c)$

– free stream, $r \rightarrow \infty$
 $Z_2 = 0. \quad (2d)$

The dimensionless initial conditions are:
 $\tau_{1(2)} = 0, Z_1 = 1, Z_2 = 0. \quad (3)$

The velocity profiles derived analytically by Hoffmann *et al.*¹⁹ were used in this work. The expressions for these velocities are:

$$V_R = \left[1 - \frac{2a}{r} + \frac{2b}{r^3} + B e^{-\kappa r} \left(\frac{2}{\kappa^4 r^3} + \frac{2}{\kappa^3 r^2} \right) \right] \cos \theta \quad (4)$$

$$V_\theta = \left[1 - \frac{a}{r} - \frac{b}{r^3} - B e^{-\kappa r} \left(\frac{1}{\kappa^4 r^3} + \frac{1}{\kappa^3 r^2} + \frac{1}{\kappa^2 r} \right) \right] \sin \theta, \quad (5)$$

where

$$a = 0.750 - B \frac{e^{-\kappa}}{2\kappa^2}, \quad b = 0.250 - B e^{-\kappa} \frac{2\kappa^2 + 4\kappa + 4}{4\kappa^4},$$

$$B = \frac{3\kappa^2(1-\alpha)}{2e^{-\kappa} \left[\kappa \left(\alpha - 1 - \frac{1}{\chi} \right) - \frac{1}{\chi} \right]},$$

$$\kappa = \sqrt{\lambda \frac{2}{1+\chi}}, \quad \chi = \frac{\mu_R}{\mu}, \quad \lambda = \frac{2\mu_R R^2}{\gamma},$$

μ is the classical dynamic viscosity coefficient of the fluid, μ_R is the vortex viscosity coefficient and γ is one of the spin gradient viscosity coefficients. In the previous relations, the parameter α characterizes the interaction between the fluid and the boundary. It takes values in the range, $0 \leq \alpha \leq 1$. The physical significance for the values of α is: (1) $\alpha = 0$ – high concentration of micro-elements on the boundary; the microstructure does not rotate relative to the boundary; (2) $\alpha = 1$ – the concentration of the particles on the boundary tends to zero; the micro-rotation is equal to the mean flow macro-rotation (one half from the vorticity). When $\alpha = 1$ or $\chi = 0$, the velocities previously presented coincide with the Stokes velocities.

The physical quantities of interest are the sphere dimensionless average temperature \bar{Z}_1 , the instantaneous local Nusselt number, Nu_θ , and the instantaneous overall Nusselt number, $Nu_{1(2)}$. Considering as driving force the difference between the instantaneous sphere average temperature and the free stream temperature, the instantaneous local and average Nu numbers are given by:

– local Nu

$$Nu_\theta = \frac{2}{\bar{Z}_1} \left(- \frac{\partial Z_2}{\partial r} \Big|_{r=1+} \right), \text{ if } \Phi \geq 1 \quad (6a)$$

$$Nu_\theta = \frac{2}{\bar{Z}_1} \left(- \frac{\partial Z_1}{\partial r} \Big|_{r=1-} \right), \text{ if } \Phi < 1, \quad (6b)$$

– average Nu

$$Nu = \frac{1}{2} \int_0^\pi Nu_\theta \sin \theta d\theta \quad (7)$$

or

$$Nu = - \frac{2}{3} \frac{d \ln \bar{Z}_1}{d \tau_1}, \text{ if } \Phi \leq 1 \quad (8a)$$

$$Nu = - \Xi \frac{2}{3} \frac{d \ln \bar{Z}_1}{d \tau_1}, \text{ if } \Phi > 1. \quad (8b)$$

The instantaneous sphere dimensionless average temperature was calculated with the relation,

$$\bar{Z}_1 = \frac{3}{2} \int_0^1 \int_0^\pi Z_1 r^2 \sin \theta d\theta dr. \quad (9)$$

METHOD OF SOLUTION

The numerical algorithm is identical to that used in²⁰ (for example). Its main components are:

- equations (1a) and (1b) are considered a single domain conjugate problem;
- discretization of the spatial derivatives with the conservative finite difference scheme (finite volume) on a vertex-centered grid;²¹
- the use of the regularization coefficient given in²² to obtain a positive definite discrete operator;
- outside and inside the sphere, the values of the discretization steps are: $\Delta\theta = \pi / 128, \pi / 256, \pi / 512$ and $\Delta r = 1 / 128, 1 / 256, 1 / 512$;
- the classical ADI algorithm developed for 2D parabolic problems was used for time integration.

The dimensionless time step was variable and changed from the start of the computation ($10^{-9} - 10^{-6}$) to the final stage ($10^{-6} - 10^{-4}$). The time integration was stopped when $\bar{Z}_1 \leq 10^{-4}$.

Two error criteria were used: (i) mesh independence of the Nu numbers calculated with relations (7) and (8a,b); (ii) the relative difference between the Nu numbers calculated with relations

(7) and (8a,b) must be smaller than 1%. In literature there are no results that can be used to validate the accuracy of the present heat transfer computations. A partial validation, presented in the next section, consists of: when Φ and Ξ tend to infinity, the asymptotic Nu value of the conjugate problem tends to the Nu value corresponding to the heat transfer from a sphere with constant temperature.

RESULTS

The numerical solutions of the mathematical model depend on the dimensionless groups Pe , Φ , Ξ and the non-Newtonian fluid parameters α , χ and λ . Creeping flow is valid for $Re < 1$. For this reason, the numerical values assumed for Pe are $Pe = 10, 100$. For small and very small values of the conductivity ratio Φ , the sphere controls the heat transfer. In this case the influence of the velocity field on the heat transfer rate is negligible. Under these conditions, the values considered for the thermal conductivity ratio, Φ , are in the range, $\Phi \geq 1$. The heat capacity ratio, Ξ , takes values from 10^{-2} to 10^2 .

Table 1

Asymptotic Nu values for creeping flow, $\Phi = 1, \lambda = 1$ and $Pe = 10, 100$

Ξ	α	χ	$Nu (Pe = 10)$	$Nu (Pe = 100)$
0.2	1	–	0.218	1.371
		0.1	0.214	1.35
		0.33	0.207	1.31
		0.66	0.199	1.26
		1	0.191	1.215
		0.2	0.211	1.33
	0.5	0.1	0.199	1.255
		0.33	0.185	1.17
		0.66	0.174	1.10
		1	0.174	1.10
		0.2	0.211	1.33
		0.33	0.199	1.255
1	1	–	0.925	2.87
		0.1	0.913	2.85
		0.33	0.888	2.805
		0.66	0.857	2.75
		1	0.803	2.71
		0.2	0.902	2.83
	0.5	0.1	0.857	2.75
		0.33	0.806	2.66
		0.66	0.767	2.58
		1	0.767	2.58
		0.2	0.902	2.83
		0.33	0.857	2.75
5	1	–	1.999	3.34
		0.1	1.993	3.33
		0.33	1.966	3.30
		0.66	1.941	3.26
		1	1.915	3.23
		0.2	1.982	3.315
	0.5	0.1	1.937	3.26
		0.33	1.892	3.194
		0.66	1.892	3.194
		1	1.848	3.14
		0.2	1.982	3.315
		0.33	1.937	3.26

The parameters α , χ , and λ , are specific parameters for micropolar fluids. Taking into consideration the discussions from model equations, the values used for α are: $\alpha = 0, 0.5$ and 1 . Following the data presented in literature, the values used for the viscosity ratio χ are: $0 \leq \chi \leq 1$. The values of the ratio μ_R/γ are considerably greater than 1 .¹⁹ Taking also into consideration the values of sphere radius, it results in $\lambda \geq 1$ or $\lambda \leq 1$. In this work, λ takes values in the range, $0.1 \leq \lambda \leq 10$.

The convective heat transfer rate depends on the non-Newtonian parameters of the micropolar fluid. Therefore, because convection is the heat transfer

dominant mechanism for long times, the quantity used to express the non-Newtonian effects on the heat transfer rate is the asymptotic value of the overall Nu number, *i.e.* $Nu(\tau \rightarrow \infty)$ (see Fig. 2).

Samples from the numerical experiments made are presented in Tables 1 – 4. This selection captures the salient features of the process. Tables 1 – 3 show the effect of micropolar (non-Newtonian) hydrodynamic parameters α and χ on the asymptotic Nu values for a constant value of λ and different values of Φ and Ξ . For given, constant values of α and χ , the effect of λ on the heat transfer rate is presented in Table 4.

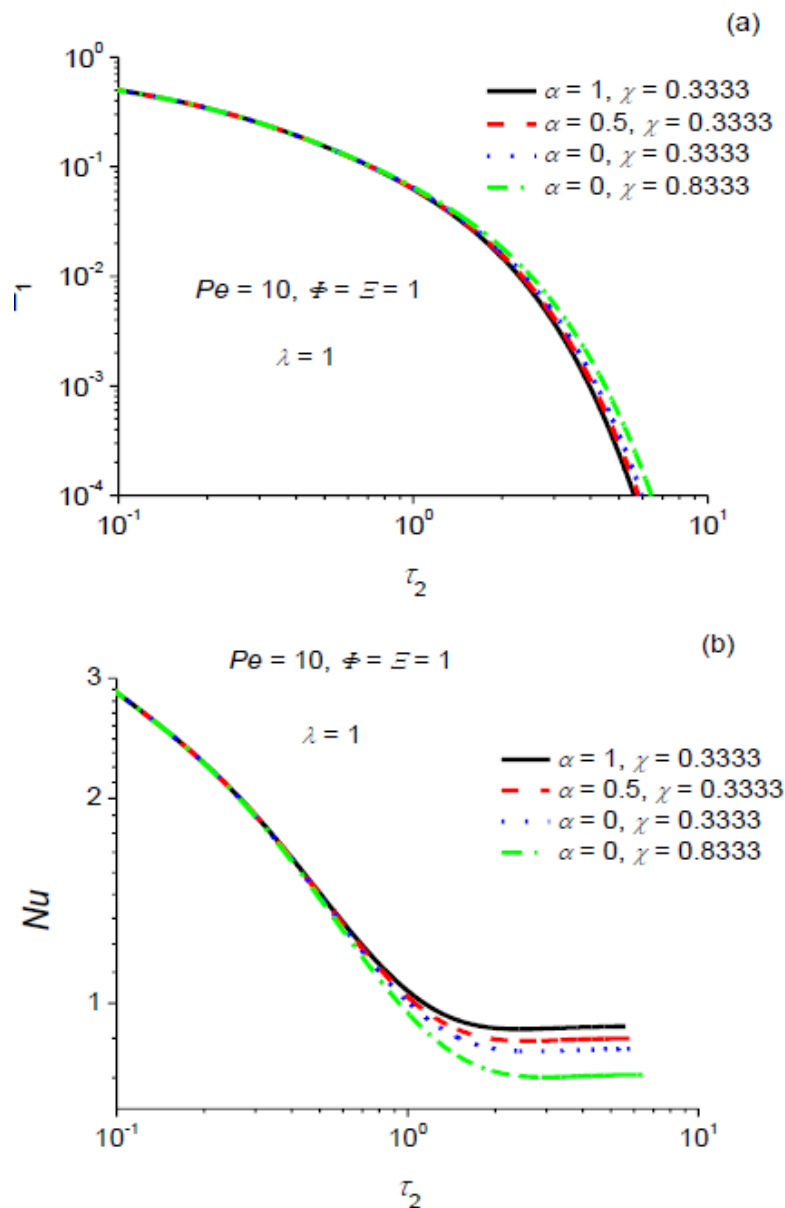


Fig. 2 – Time variation of the sphere average dimensionless temperature and instantaneous Nu number for $\Phi = \Xi = 1$, $Pe = 10$ and $\lambda = 1$; (a) $\bar{Z}_1(\tau)$; (b) $Nu(\tau)$.

Table 2
Asymptotic Nu values for creeping flow, $\Phi = 5$, $\lambda = 1$ and $Pe = 10, 100$

Ξ	α	χ	$Nu (Pe = 10)$	$Nu (Pe = 100)$
0.1	1	–	0.105	0.70
		0.1	0.103	0.69
		0.33	0.10	0.664
	0.5	0.66	0.096	0.635
		1	0.092	0.611
		0.1	0.102	0.677
	0	0.33	0.096	0.633
		0.66	0.089	0.586
		1	0.084	0.55
1	1	–	0.896	3.40
		0.1	0.885	3.36
		0.33	0.861	3.29
	0.5	0.66	0.831	3.20
		1	0.804	3.13
		0.1	0.874	3.33
	0	0.33	0.831	3.20
		0.66	0.782	3.04
		1	0.744	2.93
10	1	–	2.638	4.92
		0.1	2.632	4.89
		0.33	2.597	4.82
	0.5	0.66	2.568	4.75
		1	2.537	4.68
		0.1	2.617	4.86
	0	0.33	2.561	4.74
		0.66	2.505	4.61
		1	2.455	4.50

Table 3
Asymptotic Nu values for creeping flow, $\Phi = 100$, $\lambda = 1$ and $Pe = 10, 100$

Ξ	α	χ	$Nu (Pe = 10)$	$Nu (Pe = 100)$
1	1	–	0.875	3.39
		0.1	0.864	3.36
		0.33	0.841	3.28
	0.5	0.66	0.812	3.19
		1	0.786	3.11
		0.1	0.853	3.32
	0	0.33	0.812	3.18
		0.66	0.765	3.03
		1	0.729	2.91
100	1	–	3.192	5.58
		0.1	3.19	5.54
		0.33	3.156	5.465
	0.5	0.66	3.136	5.38
		1	3.109	5.30
		0.1	3.181	5.51
	0	0.33	3.125	5.37
		0.66	3.082	5.22
		1	3.034	5.10

Table 4
Asymptotic Nu values for creeping flow, $\alpha = 0$, $\chi = 1$ and $Pe = 10$

Φ	Ξ	λ						
		0.1	0.2	0.5	1	2	5	10
1	0.2	0.182	0.177	0.174	0.174	0.176	0.183	0.188
100	100	3.083	3.054	3.047	3.034	3.050	3.078	3.088

The numerical data presented in Tables 1 – 4 show that:

- the non-Newtonian fluid parameters that have a significant effect on the heat transfer rate are α and χ ; the decrease in α and the increase in χ decrease the heat transfer rate;
- the influence of the viscosity ratio χ on the asymptotic Nu values increases with the decrease in α ;
- the effect of λ on the heat transfer rate is not significant (for given, constant values of α and χ);
- for a fixed Φ value, the increase in Ξ decreases the influence of the non-Newtonian fluid parameters on the heat transfer rate;
- the increase in Φ (for a given, constant value of Ξ) does not change significantly the influence of the non-Newtonian fluid parameters on the asymptotic Nu values;
- the increase in Pe from 10 to 100 decreases

slightly the influence of the non-Newtonian fluid parameters on the heat transfer rate.

It must be also mentioned that the relative differences between the present values of asymptotic Nu ($\alpha = 1, \Phi = \Xi = 100$) and the values calculated with relation (3–49) from²³ are smaller than 2% (a partial validation of the present computations).

The effects of the non-Newtonian fluid parameters on the heat transfer rate discussed previously are similar to those presented in,⁵ *i.e.* the influence of α and χ on the heat transfer rate increases when $\Xi \leq 1$ and Φ increases from 1 to 100. This behavior is similar to that observed for the thermal wake phenomenon (Juncu²⁴ and the references quoted herein).

To approximate the effect of the non-Newtonian fluid parameters on the heat transfer rate, the following relation is proposed in this work:

$$Nu \left[\text{Newtonian}, Pe / \sqrt{(1 + \chi)} \right] \leq Nu (\text{Micropolar}, Pe) \leq Nu (\text{Newtonian}, Pe) \quad (10)$$

The numerical data previously presented shows that when $\alpha \rightarrow 1, Nu (\text{Micropolar}, Pe) \rightarrow Nu (\text{Newtonian}, Pe)$. For the lower limit of relation (10), it must be mentioned that the

maximum values of the relative difference between the minimum values of $Nu (\text{Micropolar}, Pe)$ and $Nu (\text{Newtonian}, Pe / (1 + \chi)^{1/2})$, *i.e.*, are around 10%.

$$\frac{Nu (\text{Micropolar}, Pe) - Nu (\text{Newtonian}, Pe / \sqrt{1 + \chi})}{Nu (\text{Micropolar}, Pe)}$$

The influence of the micropolar hydrodynamic parameters on the heat transfer mechanism was analysed in terms of local Nu number. The numerical results obtained have shown that the micropolar hydrodynamic parameters do not change the mechanism of heat transfer.

CONCLUSIONS

This work investigated numerically the unsteady conjugate heat transfer from a sphere to a micropolar fluid in creeping flow.

The numerical results obtained can be summarized as follows: (i) regardless the values of the Pe number and physical properties ratios, the parameter α and the viscosity ratio χ have the strongest influence on the heat transfer rate; the effect of λ on the heat transfer rate is not significant; (ii) the increase in Pe decreases the influence of the non-Newtonian fluid parameters on the heat transfer rate; (iii) the influence of the non-Newtonian fluid parameters on the heat transfer rate also depends on

the values of the physical properties ratio; the decrease in Ξ (for $\Xi \leq 1$) and the increase in Φ (for $\Phi \geq 1$) increases the effect of the non-Newtonian fluid parameters on the heat transfer rate.

List of symbols

- c_p Heat capacity, J / (kg · K)
- d Diameter of the sphere, $d = 2 R$, m
- k Thermal conductivity, W / (m · K)
- Nu Instantaneous average Nusselt number, dimensionless
- Nu_θ Instantaneous local Nusselt number, dimensionless
- Pe Peclet number, $Pe = U_0 d \rho c_p / k$, dimensionless
- R Radius of the sphere, m
- Re Reynolds number based on the diameter of the sphere, $Re = U_0 d \rho / \mu$, dimensionless
- r Dimensionless radial coordinate, r^* / R , in spherical coordinate system
- r^* Radial coordinate in spherical coordinate system
- t Time, s

T	Temperature, K
U_0	Velocity far away from the sphere, m / s
V_R	Dimensionless radial velocity component
V_θ	Dimensionless tangential velocity component
Z	Dimensionless temperature defined by the

$$\text{relation, } Z_{2(1)} = \frac{T_{2(1)} - T_{2,0}}{T_{1,0} - T_{2,0}}$$

Greek symbols

α	Micropolar fluid parameter, dimensionless
γ	Spin gradient viscosity coefficient, kg · m / s
χ	Viscosity ratio, dimensionless
Φ	Thermal conductivity ratio, k_1 / k_2 , dimensionless
λ	Micropolar fluid parameter, dimensionless
μ	Dynamic viscosity, kg / (m · s)
μ_R	Vortex viscosity, kg / (m · s)
θ	Polar angle in spherical coordinate system
ρ	Density, kg / m ³
τ	Dimensionless time or Fourier number, $\tau = 4$ $t k / \rho c_P d^2$
Ξ	Thermodynamic ratio, $(\rho_1 c_{P,1}) / (\rho_2 c_{P,2})$, dimensionless

Subscripts

1	Refers to the interior of the sphere
2	Refers to the exterior of the sphere
s	Refers to the surface of the sphere
0	Initial conditions

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