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COMPARISON BETWEEN THE MAXIMUM GROUND LEVEL CONCENTRACTION OF AIR POLLUTANTS AT CENTERLINE WHEN EFFECTIVE HEIGHT CONSTANT AND VARIABLE

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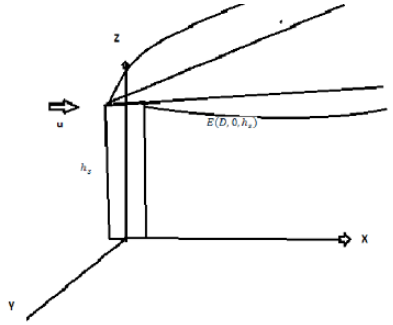
The Maximum Ground Level Concentration (MGC) of an air pollutant at centerline is obtained using the Gaussian plume model. However, the effective height *H* of the pollutant has been considered as a function of the ground level coordinate *x* and has a constant value. Extreme radius of the stack is developed. The constancy of *H* has been studied as a special case for the problem. Interesting argument about both maximum and worst concentration has been derived.



INTRODUCTION

The comparison between maximum ground level concentrations (MGLC) of air pollutants at the plume centerline under constant versus variable effective height conditions reveals significant differences in dispersion behavior and pollutant impact. When the effective height (*H*) is treated as constant – typically the sum of stack height and a fixed plume rise – the Gaussian plume model predicts a relatively stable MGLC location and magnitude. However, when *H* is modeled as a function of downwind distance (*x*) and lateral position (*y*), the plume dynamics become more sensitive to atmospheric conditions, stack parameters, and thermal buoyancy, leading to variable dispersion coefficients and shifting concentration peaks.[1,2] Studies have shown that assuming a variable effective height yields more realistic predictions, especially under unstable atmospheric conditions where plume rise fluctuates with wind speed and temperature gradients. For instance,[3] demonstrated that modeling *H* as a function of *x* and *y* provides refined estimates of pollutant spread, while Padmanabhamurty and Chakrabarty emphasized the sensitivity of MGLC to plume rise formulations and meteorological inputs.[2] These findings underscore the importance of incorporating dynamic plume behavior in air quality assessments to better capture real-world pollutant dispersion Sources.

The highest concentration of air contaminants at ground level, near the centerline of a plume, occurs

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when the plume has been effectively "turned down" to the ground by a combination of the effective stack height, atmospheric stability, and the presence of the ground surface itself. This location is often the most hazardous point for ground-level exposure and can be estimated using models like the Gaussian plume model.

The Gaussian plume model is a common mathematical tool used to predict how air pollutants from a point source, like a power plant smokestack, disperse in the atmosphere. It works by assuming pollutant concentrations across the plume follow a double Gaussian (bell curve) distribution, with the plume's shape and size determined by atmospheric conditions such as wind speed, direction, and turbulence (atmospheric stability). The model calculates pollutant concentrations by considering factors like the emission rate, stack height, wind speed, and dispersion coefficients, making it a fundamental tool for air quality assessment and emergency planning.

The (MGLC) of air pollutants are critical indicators in environmental modeling and public health risk assessments. Recent studies have refined predictive models to better estimate these concentrations under varying atmospheric conditions. For instance,[1] utilized the Gaussian Plume Model to compare actual and estimated MGLC values, revealing significant discrepancies depending on the dispersion parameter system used – highlighting the importance of accurate modeling in regulatory applications. More recently, Computational fluid dynamics (CFD) are applied to analyze how stack configurations and separation ratios influence pollution dispersion.[4] Their findings showed that altering stack separation can shift the MGLC location by up to 47% and reduce its spatial extent by as much as 46%, offering actionable insights for industrial design and urban planning. These advancements underscore the evolving sophistication of air quality modeling and its role in mitigating environmental impact.

In this work the (MGLC) of an air pollutant at centerline is obtained using the Gaussian plume model. Comparing between the effective heights H is as a function of the ground level coordinate x and has a constant value. Extreme radius of the stack is developed. The constancy of H has been studied as a special case for the problem. Interesting argument about both maximum and worst concentration has been derived.

VARIABLE OF THE EFFECTIVE HEIGHT

The effective height is given by:

$$H = h_s + \Delta h \tag{1}$$

where h_s is the stack height and Δh is the plume rise.

In this work, we take the vertical and crosswind dispersion parameters are as follows:

$$\sigma_z = ax^b \qquad and \qquad \sigma_y = cx^d \tag{2}$$

where a, b, c and d are constants depend on stability parameters.

The Gaussian Plume model in the three dimensions is as follows[5]:

$$C(x,y,z) = \frac{Q}{2\pi\sigma_y\sigma_z\bar{u}} e^{-\frac{y^2}{2\sigma_y^2}} \left\{ exp\left(-\frac{(z-H)^2}{2\sigma_z^2}\right) + exp\left(-\frac{(z+H)^2}{2\sigma_z^2}\right)\right\} \tag{3}$$

where
Q is the emission rate, $\bar{u}$ is the mean wind speed (m/s), and y, z are the crosswind and the vertical heights. Then the ground Gaussian plume model at the centerline taking H is depends on the downwind distance x as follows:

$$C(x,0,0,H) = \frac{Q}{\pi\bar{u}\sigma_y\sigma_z} exp\left(-\frac{H^2(x)}{2\sigma_z^2}\right)$$
$$C(x,0,0,H) = \frac{Q}{\pi\bar{u}acx^{b+d}} exp\left(-\frac{H^2(x)}{2a^2x^{2b}}\right) \tag{4}$$

The maximum ground level concentration $\partial C/\partial x = 0$, which gives:

$$\frac{\partial C}{\partial x} = \frac{Q}{\pi\bar{u}acx^{b+d}} exp\left(-\frac{H^2(x)}{2a^2x^{2b}}\right)\left(-\frac{H\frac{\partial H}{\partial x}}{a^2x^{2b}} + \frac{H^2b}{a^2x^{2b+1}} + \frac{b+d}{x}\right) = 0 \tag{5}$$

Multiplying Eq. (5) by $-a^2x^{2b+1}$

$$xH\frac{\partial H}{\partial x} - bH^2 - a^2x^{2b}(b+d) = 0$$

Assuming

$$H(x) = \left(x\,V(x)\right)^{b} \tag{6}$$

$$\frac{\partial H}{\partial x} = b(xV(x))^{b-1}\left(x\frac{dV}{dx} + V\right)$$

$$x^{b+1}V^{b}\left[b(xV)^{b-1}\left(x\frac{dV}{dx} + V\right)\right] - bx^{2b}V^{2b} - a^{2}x^{2b}(b+d) = 0$$

$$bx^{2b+1}V^{2b-1}\frac{dV}{dx} - a^{2}x^{2b}(b+d) = 0$$

This on integrating both sides, one gets:

$$V^{2b} = 2a^{2}(b+d)ln\frac{A}{x}$$

Using Eq. (6), one gets:

$$H^{2} = 2a^{2}x^{2b}(b+d)ln\frac{A}{x} \tag{7}$$

This equation represents a relation between the point *x* of the pollutant path and its corresponding height *H*. In order to deduce *A* we can study Eq. (7) at certain point E(D,0,h_s) on the top of the stack, where the air pollutant starts to move at velocity *u* along the x-direction, where D is the radius of the stack Fig. (1).[6] Substituting the coordinates of the point *E* into Eq. (1). Substituting the coordinates of the point *E* into Eq. (7):

$$h_s^{2} = 2a^{2}D^{2b}(b+d)ln\frac{A}{D}$$

$$ln\frac{A}{D} = \frac{h_s^{2}}{2a^{2}D^{2b}(b+d)}$$

$$A = D\,exp\left[\frac{h_s^{2}}{2a^{2}D^{2b}(b+d)}\right] \tag{8}$$

Substituting from Eq. (8) in Eq. (7), one gets:

$$H^{2} = 2a^{2}x^{2b}\left[(b+d)\ln\frac{D}{x} + \frac{h_s^{2}}{2a^{2}D^{2b}}\right] \tag{9}$$

This equation describes the pollutant height *H* in terms of the pollution *x*. Now substituting from Eq. (9) into Eq. (4), we get the desired expression for the maximum ground level concentration in the form:

$$C_{max} = \frac{Q}{\pi\bar{u}acD^{b+d}}\,exp\left[-\left(\frac{2a^{2}x^{2b}(b+d)}{2a^{2}x^{2b}}ln\frac{D}{x} + \frac{h_s^{2}}{2a^{2}x^{2d}}\right)\right]$$

$$= \frac{Q}{\pi\bar{u}acD^{b+d}}\,exp\left(-\frac{h_s^{2}}{2a^{2}D^{2d}}\right) \tag{10}$$

From Eq. (10), one can determine the extreme radius *D* for the stack that gives maximum ground concentration at the centerline as follows:

$$\frac{\partial C_{max}}{\partial D} = 0$$

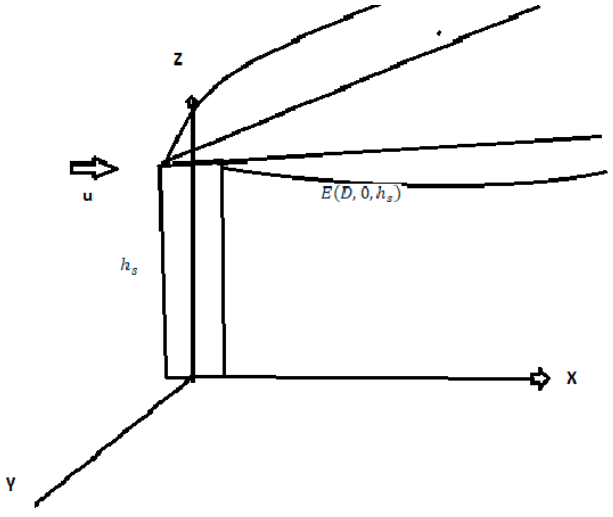

Fig. 1 – The Graph of atmospheric diffusion at the top point E(D,0,h_s).

$$\frac{\partial C_{max}}{\partial D} = \frac{Q}{\pi\bar{u}acD^{b+d}} exp\left(-\frac{h_s^2}{2a^2D^{2d}}\right)\left(\frac{2dh_s^2D^{-2d-1}}{2a^2}\right) + exp\left(-\frac{h_s^2}{2a^2D^{2d}}\right)\frac{-Q(b+d)D^{-b-d-1}}{\pi\bar{u}ac}$$

$$= \frac{Q}{\pi\bar{u}acD^{b+d}} exp\left(-\frac{h_s^2}{2a^2D^{2d}}\right)\left(\frac{h_s^2 d}{a^2D^{2d+1}} - \frac{(b+d)}{D}\right) = 0$$

$$D^{2d} = \frac{h_s^2 d}{a^2(b+d)}$$

$$D = \left(\frac{h_s^2 d}{a^2(b+d)}\right)^{\frac{1}{2d}} \quad (11)$$

$$C_{max} = \frac{Q}{\pi\bar{u}ac}\left(\frac{a^2(b+d)}{h_s^2 d}\right)^{\frac{b+d}{2d}} \exp\left(-\frac{b+d}{2d}\right) \quad (12)$$

SPECIAL CASE

If we take the effective height is **constant** (H=constant) i.e $\partial H/\partial x$=0, Eq. (5) becomes:

$$-bH^2 + a^2x^{2b}(b+d) = 0$$

$$x = \left(\frac{bH^2}{a^2(b+d)}\right)^{1/2b}$$

$$(x_{max}, y_{max}) = \left[\left(\frac{bH^2}{a^2(b+d)}\right)^{1/2b}, 0\right]$$

The maximum of the vertical and crosswind dispersion parameters as follows:

$$\sigma_{z_{max}} = ax_{max}^b = H\left(\frac{b}{b+d}\right)^{1/2} \quad (13)$$

$$\sigma_{y_{max}} = cx^d = c\left(\frac{bH^2}{a^2(b+d)}\right)^{\frac{d}{2b}} \quad (14)$$

$$C_{max} = \frac{Q}{\pi ac\bar{u}\left[\frac{bH^2}{a^2(b+d)}\right]^{\frac{b+d}{2b}}} \exp\left(\frac{-(b+d)}{2b}\right) \quad (15)$$

WORST LEVEL CONCENTRATION

Taking the effective height H equals

$$H = h_s + \frac{\lambda}{\bar{u}} \quad (16)$$

where λ is constant. Differentiating Eq. (15) with respect to $\bar{u}$, one gets as follows:

$$\lambda(b+d) = ub(h_s + \frac{\lambda}{\bar{u}})$$

$$\lambda b + \lambda d = \bar{u}bh_s + b\lambda$$

$$u_{worst} = \frac{\lambda d}{bh_s} \quad (17)$$

$$H_{worst} = h_s + \frac{\lambda bh_s}{\lambda d} = h_s + \frac{bh_s}{d} = h_s\left(\frac{b+d}{d}\right) \quad (18)$$

Substituting from two Eqns. (17) and (18) in Eq. (15), One gets as follows:

$$C_{max} = \frac{Qexp\left[\frac{-(b+d)}{2b}\right]bh_s^{-\frac{b}{d}}}{\pi ac\lambda d\left[\frac{b(b+d)}{a^2d^2}\right]^{\frac{b+d}{2b}}} \quad (19)$$

Taking the effective height H is constant then, Eq. (3) becomes at ground at centerline as follows:

$$\bar{C}_0(x) = \frac{Q}{\pi\bar{u}acx^{b+d}} exp\left(-\frac{H^2}{2a^2x^{2b}}\right) \quad (20)$$

Differentiating Eq. (20) with respect to x, one gets as follows:

$$\frac{\partial \bar{C}_0(x)}{\partial x} = \frac{Q}{\pi\bar{u}acx^{b+d}} exp\left(-\frac{H^2}{2a^2x^{2b}}\right)\left(\frac{dH^2}{c^2x^{2d+1}} - \frac{b+d}{x}\right) = 0$$

$$\frac{b+d}{x} = \frac{dH^2}{c^2x^{2d+1}}$$

$$x_{max} = \left(\frac{H^2 d}{c^2(b+d)}\right)^{1/2d} \quad (21)$$

Then the maximum concentration if the effective height is constant is as follows:

$$\bar{C}_{omax} = \frac{Q}{\pi\bar{u}ac}\left[\frac{c^2(b+d)}{H^2 d}\right]^{\frac{b+d}{2d}} exp\left(-\frac{(b+d)}{2d}\right) \quad (22)$$

Note that for particular case of b = d (*e.g.* b = d = 0.5) according to the Fickian diffusion theory, and b = d = 1 according to the statistical theories, one can show that the maximum g.l.c. occurs where $H = \sqrt{2}\,\sigma_z$ and

$$\bar{C}_{0max} = \left(\frac{2c}{\pi ae}\right)\frac{Q}{\bar{u}H^2} = \frac{cQ}{2\pi ae\bar{u}h_s^2} \qquad (23)$$

Table 1

Parameters value in the formulas *a*, *b*, *c* and *d* [7]

Stability	*a*	*b*	*c*	*d*
A-B	0.01	1.54	1.46	0.71
C	0.04	1.017	1.52	0.69
D	0.09	0.95	1.36	0.67
E-F	0.40	0.67	0.75	0.70

Table 2

Parameters value for Brookhaven National Laboratory (BNL) for the formulas *a*, *b*, *c* and *d* [8]

Stability	*a*	*b*	*c*	*d*
A-B (B_2)	0.41	0.91	0.40	0.91
C (B_1)	0.33	0.86	0.36	0.86
D (C)	0.22	0.78	0.32	0.78
E-F(D)	0.06	0.71	0.31	0.71

Table 3

The values of the wind speed through all stabilities[9]

Stability	$\bar{u}$ (m / sec)
A-B (B_2)	4
C (B_1)	4
D (C)	5.8
E-F(D)	3.8

Table 4

Shows the maximum concentrations in all stabilities from[7,8]

Stability	Essa and Hanaa		Hanna *et al.*	
	H Variable	H constant	H variable	H constant
	$C_{max}/Q \times 10^3 s/m^3$	$C_{0max}/Q \times 10^5 s/m^3$	$C_{max}/Q \times 10^3 s/m^3$	$C_{0max}/Q \times 10^5 s/m^3$
A-B (B_2)	2.47	0.93	0.086	2.01
C (B_1)	2.38	1.37	0.061	2.01
D (C)	1.48	0.98	0.013	1.39
E-F(D)	0.40	2.21	4E-07	2.11

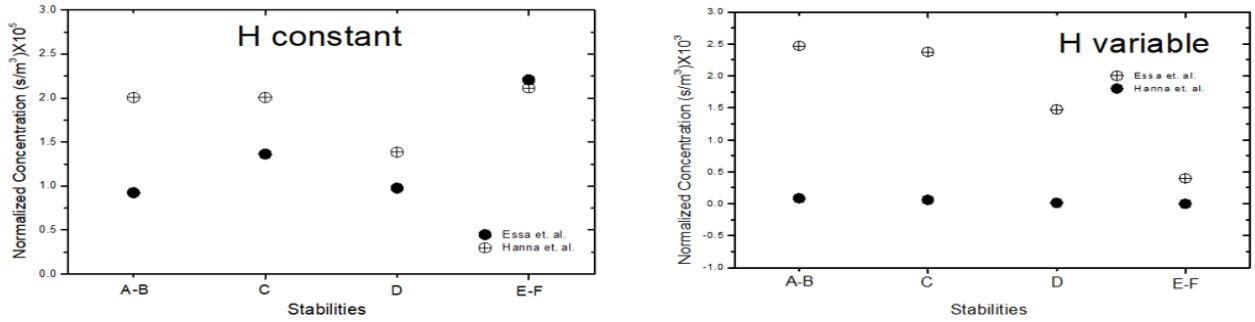

Fig. 2 – Shows that the normalized concentrations via all stabilities when H is variable and constant of Iodine-135.

Figure 2 explains the maximum concentration of I^{135} taking the parameters variables *a*, *b*, *c* and *d* from,[8,7] we find that when the effective height *H* is variable, we obtain that the values of maximum concentrations from Essa and Hanaa are larger than its values from.[8] But, when *H* is constant, the values of maximum concentrations from[8] are larger than its values from.[7]

CONCLUSIONS

When the effective height H is variable, we obtain that the values of maximum concentrations from Essa *et al*. are larger than its values from.[9] But, when H is constant, the values of maximum concentrations from[8] are larger than its values from.[7]

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